# NOISE IN QUANTUM MECHANICS

## A NEW APPROACH

PREHISTORY

HISTORY

PRESENT

REVOLUTION

FUTURE

EXAMPLE

CONCLUSIONS

# $\boxed{\text{PREHISTORY}}$ Classical

Brownian particle:

$$m\dot{v} = -\gamma v + \xi(t) \qquad \text{Langevin force}$$

$$\langle \xi(t) \rangle = 0 \qquad \langle \xi(t)\xi(t') \rangle = C\,\delta(t-t')$$

Electronic circuits:

$$R\frac{dQ}{dt} = -\frac{Q}{C} + \xi(t)$$

"$\xi(t)$ is added whenever it leads to useful results."

However, in case of nonlinear resistance,

$$R\frac{dQ}{dt} = -f\left(\frac{Q}{C}\right) + \xi(t)$$

$$R\frac{d\langle Q \rangle}{dt} = -\left\langle f\left(\frac{Q}{C}\right)\right\rangle \neq -f\left(\frac{\langle Q \rangle}{C}\right) - \frac{\langle (Q-\langle Q \rangle)^2 \rangle}{2C^2} f''\left(\frac{\langle Q \rangle}{C}\right) - \dots$$

(D.K.C. MacDonald)

Worse: $\qquad \dot{x} = f(x) + g(x)\,\xi(t)$

(Itô – Stratonovich)

$\boxed{\text{HISTORY}}$ Quantum Langevin equation

$$\dot{X} = -i[H, X] + \Xi(t)$$

How to specify the operator function $\Xi(t)$ ?

(Kač & Benguria)

<u>Conclusion</u> :

Include explicitly the noise source

Down with Langevin !

Not a deus ex machina to create irreversibility or to destroy coherence!

$\boxed{\text{PRESENT}}$ Add bath

$$\underset{\substack{\uparrow \\ \text{total}}}{H_T} = \underset{\substack{\uparrow \\ \text{system}}}{H_S} + \underset{\substack{\uparrow \\ \text{bath}}}{H_B} + \lambda \underset{\substack{\uparrow \\ \text{interaction}}}{H_I}$$

$$\rho_T(t) = e^{-itH_T} \rho_T(0)\, e^{itH_T} \equiv e^{-it\mathcal{L}_T} \rho_T(0)$$

$$\rho_S(t) = \operatorname{Tr}_B \rho_T(t).$$

(†) Assume initial state uncorrelated $\rho_T(0) = \rho_S(0) \otimes \rho_B^{eq}$

(††) For small $\Delta t$ to second order in $\lambda$

$$\rho_S(\Delta t) = \rho_S(0) + \Delta t\, \mathcal{L}_S \rho_S(0) + $$
$$+ \Delta t\, \lambda^2 \int_0^{\Delta t \to \infty} d\tau \left\{ \operatorname{Tr}_B \mathcal{L}_I e^{\tau \mathcal{L}_0} \mathcal{L}_I e^{-\tau \mathcal{L}_0} \rho_B^{eq} \right\} \cdot \rho_S(0)$$

(†††) Conclude (Zwanzig)

$$\frac{d\rho_S(t)}{dt} = \left[ \mathcal{L}_S + \lambda^2 \int_0^\infty d\tau \left\{ \quad \right\} \right] \rho_S(t)$$

## $\boxed{\text{REVOLUTION}}$

Minimum requirements : At all times

(i) $\mathrm{Tr}_s\, \rho_s = 1$

(ii) $\rho_s^\dagger = \rho_s$

(iii) $\rho_s$ positive definite

Consequently, if $\dot{\rho}_s = \mathcal{L} \rho_s$

conditions on $\mathcal{L}$ (Kossakowski, Lindblad)

Our result violates (iii) : negative probabilities

Culprit : Repeated Randomness Assumption

$\boxed{\text{FUTURE}}$ New approach

Total equilibrium

$$\rho_T^{eq} = e^{-\beta H_T}/Z \qquad Z = \operatorname*{Tr}_T e^{-\beta H_T} \qquad H_T = H_S + H_B + \lambda H_I$$

For any operator $F$ on $S$ : $\langle F \rangle^{eq} = \operatorname*{Tr}_T F \rho_T^{eq}$

$$\langle F(t) F(t+\tau) \rangle^{eq} = \operatorname*{Tr}_T \{ F\, F(\tau)\, \rho_T^e \}$$

contains all required information and is exact.

More general: for two operators $G, F$ acting on $S$

$$\langle G(t) F(t+\tau) \rangle^{eq} = \operatorname*{Tr}_T \{ G\, F(\tau)\, \rho_T^e \}$$

Explicit calculation to order $\lambda^2$ gives

$$\begin{aligned}
\langle G\,F\{\tau\}\rangle^e_\lambda &= \langle G\,F(\tau)\rangle^e_0 \left\{ \boxed{1 - \lambda^2 \int_0^\beta d\beta' \int_0^{\beta'} d\beta'' \,\langle H_I(-i\beta') H_I(-i\beta'')\rangle^e_0} \right\} \\
&- \lambda^2 \int_0^\tau dt' \int_0^{t'} dt'' \,\langle G\,[H_I(t''),[H_I(t'),F(\tau)]]\rangle^e_0 \\
&\boxed{\begin{aligned} &+ \lambda^2 \int_0^\beta d\beta' \int_0^{\beta'} d\beta'' \,\langle H_I(-i\beta') H_I(-i\beta'') G\,F(\tau)\rangle^e_0 \\ &- i\lambda^2 \int_0^\beta d\beta' \int_0^\tau dt' \,\langle H(-i\beta') G\,[H_I(t'),F(\tau)]\rangle^e_0 . \end{aligned}}
\end{aligned}$$

Pre-revolutionary calculation omits integrals on $\beta$

## $\boxed{\text{EXAMPLE}}$

$$H_T = \tfrac{1}{2}(P_0^2 + \Omega^2 Q_0^2) + \tfrac{1}{2}\sum_k (P_k^2 + k^2 Q_k^2) + Q_0 \sum \lambda_k Q_k$$

Bath frequencies dense and $\lambda_k$ smooth

$$Q_k = \sum_\nu X_{k\nu}\, q_\nu \qquad P_k = \sum X_{k\nu}\, p_\nu$$

$$H_T = \tfrac{1}{2}\sum_\nu (p_\nu^2 + \omega_\nu^2 q_\nu^2)$$

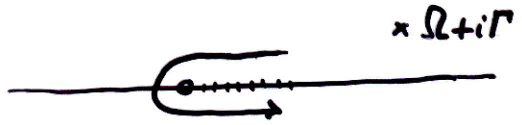
$\omega_\nu^2$ are zeroes of $G(\omega) \equiv \omega^2 - \Omega^2 - \sum_k \frac{\lambda_k^2}{\omega^2 - k^2}$

Resulting correlation function:

$$\langle Q_0(0)\, Q_0(t)\rangle^{eq} = \frac{1}{2\pi i}\oint \frac{d\omega}{G(\omega)}\left\{\coth\frac{\beta\omega}{2}\cdot\cos\omega t + i\sin\omega t\right\}$$

Pole at $\Omega + i\Gamma$

yields standard result:




Complex $\omega$ plane

$$\langle Q_0(0)\, Q_0(t)\rangle^{eq} = \frac{1}{2\Omega} e^{-\Gamma t}\left\{\coth\frac{\beta\Omega}{2}\cdot\cos\Omega t + i\sin\Omega t\right\}$$

Pole at $\omega = \frac{2\pi}{\beta}$ yields $\exp\left[-\frac{2\pi}{\beta}t\right]$

## CONCLUSIONS

Do not add a random force without the source from where it originates.

Do not use the word "random" as a means to cover your ignorance.

Do not take the unrealistic initial condition in which your system is uncorrelated with the source of fluctuations.