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Josephson tunneling as a probe of the vortex-glass state

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Josephson tunneling between superconducting electrodes in the vortex-glass state is considered. The Josephson coupling energy and phase offset are stochastic, the typical value of the former scaling as the Edwards-Anderson order parameter, Larkin-Ovchinikov length, and square root of the junction area. The influence of magnetic flux through the junction is explored, and magneto-fingerprints, reminiscent of universal conductance fluctuations, are predicted. Properties of SQUIDs are also examined, and it is concluded that Josephson phenomena may provide a useful probe of the vortex-glass state.

It has recently been proposed that, owing to the presence of quenched disorder, high-temperature superconductors penetrated by a magnetic field may exhibit a novel equilibrium state, the so-called vortex-glass state.[1,2] In this state (and with a particular choice of gauge), the equilibrium expectation value $\langle\psi(\mathbf{x})\rangle$ of the complex superconducting order parameter $\psi(\mathbf{x})$ (Ref. 3) would vary randomly with position $\mathbf{x}$ but would be frozen in time. Thus, vortex-glass order could be diagnosed using a version of the Edwards-Anderson order parameter,[1,4]

$$Q \equiv \frac{1}{V}\int_V d^3x\,|\langle\psi(\mathbf{x})\rangle|^2, \tag{1}$$

where V denotes the volume of the sample. Equivalently, such order could be diagnosed using the correlator[2]

$$G(\mathbf{x}) \equiv \frac{1}{V}\int_V d^3x'\,|\langle\psi^*(\mathbf{x}')\,\psi(\mathbf{x}'+\mathbf{x})\rangle|^2, \tag{2}$$

in the limit $|\mathbf{x}|\to\infty$ which, like the Edwards-Anderson order parameter, is zero in the normal state and nonzero in the vortex-glass state. Thus, the vortex-glass state revives the possibility of a bulk thermodynamic phase transition to a state with zero linear electrical resistivity and off-diagonal long-range order for three-dimensional superconductors with quenched disorder at low temperatures in the presence of a penetrating magnetic field.

The purpose of this paper is to explore some consequences of the Josephson coupling of two independent electrodes, each of which is in the vortex-glass state, and to examine whether Josephson phenomena[5] provide a useful probe of the vortex-glass state. Our aims are to make a simple estimate for the typical coupling energy (or, equivalently, the typical critical current) of a Josephson junction fabricated from electrodes in the vortex-glass state, to analyze the influence of external magnetic flux through such a junction and estimate the magnitude of certain characteristic magnetic field strengths, and to consider the properties of flux-linked two-junction SQUIDs fabricated from vortex-glass state leads. Our estimates will be given in terms of—*inter alia*—the Edwards-Anderson order parameter and the Larkin-Ovchinikov (LO) length (see below), which are characteristics of the vortex-glass state that are commonly regarded as difficult to observe.

We envisage the electrodes of the junction as having been cooled to a low temperature in the presence of a magnetic field, so that the vortex-glass phase is well established, and the irreversible dynamics of the vortices may be neglected. In other words, we envisage the vortices as being permanently frozen into a certain random configuration, so that the global phase associated with the superconducting order parameter is the only important remaining degree of freedom in each electrode. Thus, for the left (ℓ) electrode we consider the equilibrium state in which $\langle\psi_\ell(\mathbf{x})\rangle = \langle\psi_\ell(\mathbf{x})\rangle_0$, together with the family of equilibrium states generated from it by a global phase change, for which $\langle\psi_\ell(\mathbf{x})\rangle = e^{i\phi_\ell}\langle\psi_\ell(\mathbf{x})\rangle_0$, and similarly for the right (r) electrode. Of course, the choices of reference states (i.e., the phase of $\langle\psi_\ell(\mathbf{x})\rangle_0$ and $\langle\psi_r(\mathbf{x})\rangle_0$) are arbitrary. The (kinematically independent) phase variables ϕ_ℓ and ϕ_r are coupled via the junction.

We consider a Josephson junction of square cross-section having thickness D and area A ($\equiv L^2$). The electrodes, being independent realizations of the vortex-glass state at low temperature in samples with independent quenched disorder, can be regarded as presenting to each other independent, frozen, random superconducting order having the following property: on length scales smaller than a characteristic length L_P (i.e., the Larkin-Ovchinikov length) the order parameter reflects the presence of a crystalline array of vortices (i.e., an Abrikosov flux-line lattice), whereas on length scales larger than L_P the crystalline order of the vortices is suppressed.[8,9] Equivalently, as the superconductors in question are strongly type II,[6,7] we may—in the vortex-glass state—regard the superconducting order parameter as being essentially constant in magnitude (except at vortex lines, on which it vanishes), its phase varying predictably and periodically on length scales smaller than L_P, but randomly on larger length scales.

Our starting point is the following model for the Josephson coupling energy (i.e., the energy associated with the coupling of the two electrodes) when the electrodes are in the states labeled by the phases ϕ_ℓ and ϕ_r:

$$E(\phi_\ell, \phi_r) = -\frac{J}{2}\int_A d^2x\, e^{-i\phi_\ell}\,\langle\psi_\ell^*(\mathbf{x})\rangle_0\, e^{i\phi_r}\,\langle\psi_r(\mathbf{x})\rangle_0 + \text{c.c.}, \tag{3}$$

where the integral is taken over the area of the junction. Here, J is the microscopic coupling parameter, which (at least in the vicinity of the phase transition temperature T_c) may be estimated via $(2\pi/\Phi_0)J \sim (\pi/4)(eRk_BT_c)^{-1}$, in which $\Phi_0 \equiv h/2e$ is the superconducting flux quantum, $-e$ is the electronic charge, and R^{-1} is the normal-state conductance per unit area of the junction. Introducing the coupling energy E_J and phase offset Φ_J, defined by

$$J\int_A d^2x\,\langle\psi_\ell^*(\mathbf{x})\rangle_0\,\langle\psi_r(\mathbf{x})\rangle_0 = E_J\, e^{i\Phi_J}, \tag{4a}$$

the Josephson coupling energy becomes

$$E(\phi_\ell, \phi_r) = -E_J\,\cos(\phi_r - \phi_\ell + \Phi_J). \tag{4b}$$

[The critical current I_J is related to E_J by $I_J = (2\pi/\Phi_0)E_J$.] In common with conventional junctions (i.e., junctions for which the electrodes are in the Meissner state) the Josephson coupling energy is a periodic function of the phase difference $(\phi_r - \phi_\ell)$ between the states of the two electrodes. Thus, a Josephson junction fabricated from superconducting electrodes in the vortex-glass state will exhibit versions of the dc and ac Josephson effects.[10] However, in contrast with conventional junctions, the parameters E_J and Φ_J are *stochastic parameters, dependent upon the precise configuration of the vortices in the electrodes*. The fact that the (global) gauge symmetry is not *macroscopically* broken is responsible for the absence of an obvious preferred relative phase between the electrodes.

To estimate the typical magnitude of the coupling energy E_J we use a simple "random walk" argument. We assume that $A \gg L_P^2$, so that we may regard the area of each electrode as comprising a number of order A/L_P^2 ($\gg 1$) LO regions, each of approximate area L_P^2. Within each LO region the Abrikosov flux lines will be effectively crystalline, but the flux-line lattices of distinct LO regions will be randomly configured with respect to each other. (It may be useful to regard each LO region as the analogue of a single magnetic moment in a spin glass.) Next, assemble the total junction coupling energy E by adding the contributions from each LO region. Then, over one such region the order parameter in each electrode will not be constant, but it will vary deterministically, so that the optimal coupling energy associated with that region will be of order JL_P^2Q (although perhaps significantly reduced by a geometrical factor due to spatial misalignment of the flux-line lattices).[11] However, although the relative *global* phase of the electrodes may be optimal [i.e., $E_J = -\min_{\phi_\ell,\phi_r} E(\phi_\ell, \phi_r)$], the relative phases of the superconducting order parameters in adjacent LO regions on either side of the junction will be random. Thus, collecting contributions from the entire junction, we expect that E_J will be a stochastic parameter, typically having a value of order

$$E_J \sim JL_P^2Q\sqrt{A/L_P^2} \sim JQL_P\sqrt{A}. \tag{5}$$

Equivalently, $I_J \sim \big(L_P\sqrt{A}/R\big)\big(Q/ek_BT_c\big)$. It is noteworthy that the typical value of E_J scales as the *square root* of the geometric area of the junction and as the first power of the Edwards-Anderson order parameter and of the Larkin-Ovchinikov length.[12] The phase offset Φ_J is also a stochastic parameter, uniformly distributed over the interval $(0, 2\pi)$, as (by gauge invariance) it must be.

In any given experiment, the value of E_J (and of Φ_J) that is actually realized will depend on the precise positions of the impurities that pin the vortices, as well as the detailed configuration that the system of vortices happens to adopt upon freezing. Thermal cycling to temperatures above the superconducting transition temperature will generate new frozen vortex (and possibly impurity) configurations, and thus lead to new realizations from the ensemble of possible values of E_J (and Φ_J). New realizations may also be obtained *within* the vortex-glass state, by cycling through temperatures at which vortex rearrangement occurs. The changes in E_J (and Φ_J) reflect changes in the underlying vortex configuration, and thus provide information on the collection of possible equilibrium states.

Until now, we have envisaged the magnetic field as being excluded from the junction. What are the implications of relaxing this condition? Consider a weak homogeneous magnetic field of magnitude H, threading the junction, and assume that it will not cause a significant change in the vortex configuration in either electrode.[13] The field will, however, introduce an electromagnetic phase shift[6,7] that varies deterministically across the area of the junction, so that $E_J(H)$ and $\Phi_J(H)$ are now defined by

$$J\int_A d^2x\,\langle\psi_\ell^*(\mathbf{x})\rangle_0\,\langle\psi_r(\mathbf{x})\rangle_0\,\exp\left(i\frac{2\pi}{\Phi_0}\int_{\ell\to r}\mathbf{A}\cdot d\mathbf{l}\right) = E_J(H)\,e^{i\Phi_J(H)}, \tag{6}$$

where $\mathbf{A}(\mathbf{x})$ is the usual vector potential, and the integral in the exponent is taken across the junction from the left to the right electrode. Provided that $H \ll H^{(2)} \equiv \Phi_0/DL_P$ (i.e., provided that the magnetic field is not strong enough to cause a significant phase shift over any typical LO region) the primary effect of the magnetic field will be to produce a relative rephasing of the contributions from the collection of LO regions to the integral, Eq. (4a), that define E_J and Φ_J. Thus, new elements from the ensemble of possible values of E_J and Φ_J will be achieved whenever roughly one flux quantum is added to the junction: there will be an apparently stochastic (but reproducible) variation of E_J and Φ_J with junction flux (i.e., a magneto-fingerprint). The scale of magnetic fields over which E_J and Φ_J remain correlated is of order $H^{(1)} \equiv \Phi_0/DL$.[14]

Now envisage increasing the strength H of the magnetic field in the junction beyond values of order $H^{(1)}$, continuing to assume that the vortex configuration remains frozen and unaffected by the field.[13] Initially, E_J and Φ_J will continue to vary stochastically, on the field scale $H^{(1)}$, until the magnetic field approaches a value of order $H^{(2)} \equiv \Phi_0/DL_P = (L/L_P)H^{(1)}$. At this larger scale, roughly L/L_P quanta of flux penetrate the junction, so that regions of width L_P contain roughly one flux quantum, and there is, therefore, a significant phase shift over each LO region. Thus, the contribution to E_J from each LO region begins to be suppressed and, while E_J and Φ_J continue to vary stochastically, the typical value of E_J diminishes towards zero.

The stochastic variation of the phase offset $\Phi_J(H)$ with the magnetic field H in the junction will have a noteworthy consequence. Suppose that the junction is maintained in, say, a zero-current state, and that the voltage across it is continuously monitored, as H is increased at a constant rate. Then, in order for the current to remain zero, the phase difference $(\phi_r - \phi_\ell)$ will have to respond, so as to compensate for the instantaneous value of $\Phi_J(H)$. This phase response will cause an apparently stochastically varying voltage to develop across the junction. Equivalently, if the junction is maintained in a zero-voltage state, so that the phase-difference $(\phi_r - \phi_\ell)$ remains constant, then as H is increased the current will vary, apparently stochastically, due to the stochastic variation of $\Phi_J(H)$ and $E_J(H)$.

Consequences of the vortex-glass state, particularly the stochastic variation of Φ_J with H, also emerge in the context of two-junction SQUID rings[15] fabricated from a pair of superconducting leads, each in the vortex-glass state.[16] Consider the maximum supercurrent I_S through the SQUID, and focus on its dependence on an external magnetic field, which we suppose to be directed perpendicular to the plane of the SQUID ring. Once again, we assume that the external magnetic field can be varied without altering the vortex configuration in the superconducting leads.[13] We suppose that the area Σ of the SQUID ring is considerably larger than the area of the junctions (projected onto the plane perpendicular to the magnetic field) LD.

For a given frozen vortex configuration, and for $H \ll H^{(1)}$ (so that the flux penetrating the junctions may be neglected), the maximum supercurrent I_S through the SQUID will oscillate as a function of the magnetic field with a period $H^{(0)} \equiv \Phi_0/\Sigma$, in common with the field dependence of the maximum supercurrent through a conventional SQUID.[15] However, in contrast with a conventional SQUID, the maxima of I_S will not be located at integral multiples of $H^{(0)}$, but instead will all be offset by a common stochastic shift, depending on the precise frozen configurations of the vortices in the leads (through the values of Φ_J at the junctions). This stochastic shift arises because, owing to the random configuration of the vortices in the vortex-glass state, it will generically not be true that the junctions will *both* carry their maximum supercurrent for the *same* value of the relative phase of the two leads $(\phi_r - \phi_\ell)$. In addition, due to the likelihood that the junctions will have unequal coupling energies, the minima of I_S (as a function of H) will not typically vanish (in common with a conventional Meissner SQUID comprising nonidentical junctions), and instead, like the maxima, will scale as $(2\pi/\Phi_0)E_J$.[17]

In the conventional case, when the strength of the magnetic field reaches the scale $H^{(1)}$, the SQUID interference oscillations of I_S become convoluted with the single-junction diffraction variations. This will not happen for the vortex-glass SQUID ring. Instead, the single-junction couplings will begin to vary stochastically with the field, on the scale $H^{(1)}$, as will the phase offsets. Thus, superposed on the more frequent *oscillations*, of period $H^{(0)}$, will be a slower, *stochastic*, H dependence of the amplitude and phase of I_S, having a correlation scale of order $H^{(1)}$. For field strengths H of order $H^{(2)}$ the suppression of the coupling at each junction will lead to a suppression of the typical value of I_S. The possibility that the vortex-glass state fails to have true long-range order, instead remaining rigid only over a finite (albeit long) distance, could be probed using SQUID rings with a range of circumferences. Then the characteristic SQUID response would be lost for rings of sufficiently large circumference.

To summarize, we have explored some consequences of the Josephson coupling of superconducting electrodes in the vortex-glass state. We have argued that the critical Josephson current I_J is a stochastic variable, and have estimated its typical value to be of order $\left(L_P\sqrt{A}/R\right)\left(Q/ek_BT_c\right)$, where A is the junction area. We have considered the implications of a magnetic field H threading the junction, and have observed that I_J and the phase offset Φ_J vary stochastically as a function of H, yielding magneto-fingerprints that are reminiscent of universal conductance fluctuations. For the case of a single junction, we have identified two characteristic scales of magnetic field: $H^{(1)} \equiv \Phi_0/DL$ (where D is the thickness of the junction and L is its breadth), which describes the field scale over which stochastic variations of $I_J(H)$ and $\Phi_J(H)$ remain correlated; and $H^{(2)} \equiv \Phi_0/DL_P$, at which considerable suppression of $I_J(H)$ begins, and which can be regarded as providing a measure of L_P. We have also explored some properties of two-junction SQUID rings, the leads of which are in the vortex-glass state. The SQUID ring configuration provides access to the (guage-invariant) difference between stochastic phase offsets occurring at a pair of vortex-glass Josephson junctions. We have considered the maximum current I_S through such a SQUID ring, focusing on the dependence of I_S on magnetic field. I_S oscillates with H, with a period set by a third scale of magnetic fields $H^{(0)} \equiv \Phi_0/\Sigma$ (where Σ is the area of the SQUID ring), the amplitude and phase of oscillation varying stochastically over the (larger) field scale $H^{(1)}$. We conclude that Josephson phenomena can serve as a useful probe of certain quantities that characterize the vortex-glass state, such as the Edwards-Anderson order parameter Q and the Larkin-Ovchinikov length L_P.

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[2]D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).

[3]We adopt the convention that ψ is the complex-valued gap function, and therefore has dimensions of energy. It can, of course, be rendered dimensionless by normalization with respect to, say, the magnitude of the gap at zero temperature.

[4]See, e.g., K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).

[5]See, e.g., B. D. Josephson, Adv. Phys. **14**, 419 (1965); P. W. Anderson, *Basic Notions of Condensed Matter Physics* (Benjamin, Reading, 1984); E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics* (Pergamon, Oxford, 1980), Vol. 2 ; and Refs. 6 and 7.

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[9]At least two extreme experimental configurations can be envisaged: either (i) no vortices pass into the junction from either electrode and, instead, the vortices terminate on the superconductor-vacuum boundaries of the electrodes; or (ii) all vortices arrive at the junction, roughly perpendicular to the junction. We thank D. J. Van Harlingen (private communication) for remarking on the virtues of the latter case.

[10]Similar results are anticipated for Josephson junctions between electrodes, one of which is in the vortex-glass state, the other being in, say, the Meissner state.

[11]We assume that the order parameter at the junction represents, at least moderately faithfully, the order parameter in the bulk.

[12]This estimate for the typical value of E_J can also be obtained in the following more formal way. Suppose that a voltage V_J is applied to the junction, causing the phase difference $(\phi_r - \phi_\ell)$ to increase at a rate given by the Josephson frequency $\omega_J = 2eV_J/\hbar$. Then the time average (taken over one period $T = h/2eV_J$) of the coupling energy E vanishes. However, the time average of E^2 does not vanish, and is equal to $E_J^2/2$. Averaging over the quenched disorder (which we denote by $[\cdots]$), noting that the disorder in the two electrodes is independent, and characterizing the vortex-glass order by the correlator $|[\langle\psi^*(\mathbf{x})\rangle_0\,\langle\psi(\mathbf{x}')\rangle_0]| \sim Q\exp\big(-|\mathbf{x}-\mathbf{x}'|/L_P\big)$, leads to the given estimate of the typical value: $E_J \sim JQL_P\sqrt{A}$.

[13]In principle, altering (e.g., removing) the magnetic field can be expected to cause a change in the equilibrium vortex configurations of the electrodes. However, we assume that, for small alterations and at low temperatures, changes of the vortex configurations occur only over very long time scales. On shorter time scales the vortex configurations should remain essentially unchanged.

[14]The mechanism of producing stochastic dependences on the magnetic field via the superposition of random phases is, of course, analogous to that producing conductance fluctuations in mesoscopic samples of disordered conductors; see, e.g., P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).

[15]See, e.g., Ref. 6, Sec. 22.8, or Ref. 7, Sec. 6.3.

[16]A single-junction SQUID ring might also have interesting properties. We thank A. J. Leggett (private communication) for this remark.

[17]We have ignored complications that may arise due to the self-inductance of the SQUID ring. Self-inductance may itself lead to an offset of $I_S(H)$, independent of the intrinsic stochasticity arising from the vortex-glass state.