RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 49, NUMBER 16 15 APRIL 1994-II

Quantum Hall effect in Coulomb drag: Interlayer friction in strong magnetic fields

Efrat Shimshoni
Beckman Institute, 405 North Mathews Avenue, Urbana, Illinois 61801
and Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

S. L. Sondhi
Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801
(Received 8 February 1994)

We study the Coulomb drag between two spatially separated electron systems in the quantum Hall regime. At a fixed temperature, the drag is peaked near the transitions between plateaus. The temperature dependence of the drag is a probe of the dynamics of the Hall system in a frequency and wave-vector regime inaccessible to transport measurements in a single layer. Most strikingly, in the transition regions it directly measures an exponent η, discussed by Chalker, that is characteristic of the fractal structure of the critical eigenstates.

The progress in the fabrication of semiconductor heterostructures has led to the discovery of a variety of novel phenomena in low-dimensional electron systems. Perhaps the most striking of these is the quantum Hall effect (QHE), observed in two-dimensional electron systems (2DES's).[1] More recently, interest has focused on systems with two or more electron systems in proximity which exhibit further phenomena including, among others,[2] the Coulomb drag predicted by Pogrebinskii and by Price.[3] This paper reports work at the intersection of these streams: we investigate the drag in strong magnetic fields and show that it could serve as a particularly interesting probe of the dynamics of *single* layer states in the quantum Hall regime.

Coulomb drag is the phenomenon whereby a current flowing in one system induces a current or voltage in another, nearby system, even though the two do not exchange particles. The geometry of interest is depicted in Fig. 1. If the upper system is in an open circuit, the drag is measured by the transresistivity ρ_t:

$$\rho_t = \mathcal{E}_u / j_d \,, \qquad (1)$$

where $\mathcal{E}_u$ is the parallel electric field induced in U in response to a current density j_d established in D. Recent experiments, all in zero magnetic field, have successfully measured the drag in heterostructures involving a 2DES, separated from another electron[4,5] or hole[6] system. Solomon *et al.*[4] observed the drag in a three-dimensional electron gas (3DEG) in a GaAs gate electrode, while Gramila *et al.*[5] and Sivan *et al.*[6] studied double layer structures at lower temperatures. These have been accompanied by a number of theoretical studies.[7–10]

The physics of the drag is quite intuitive for it is much like ordinary friction. While a perfectly uniform current carrying layer does not exert a Coulomb force on its neighbor, an inhomogeneous one creates a "corrugated" potential which pushes the electrons in the other in the direction of net current flow. In systems which are uniform on average, the role of inhomogeneities is played by long lived density fluctuations, i.e., the drag is sensitive to the relative abundance of density fluctuations at low frequencies. More formally, an expression for ρ_t [Eq. (6)] derived by Zheng and MacDonald[10] shows that at low temperatures T it probes the density response functions $\mathrm{Im}\chi_{u,d}(q,\omega)$ of the systems for $\omega \to 0$ at finite q (*in contrast* to dc transport measurements in a single layer, which probe the opposite limit[11]). This ties in nicely with the QHE, as density fluctuations which also play a special role in its physics. The observed plateaus in the Hall conductivity σ_{xy} and the associated vanishing of the longitudinal conductivity σ_{xx} reflect the presence of incompressible states deep inside them. Complementary, compressible physics is believed to prevail in the transition regions between plateaus, where one expects either critical "metallic" states reflecting delocalization[12] or, as recently suggested[13] for cleaner systems, "Fermi-liquid" (FL) states; however, these regions are not nearly as well understood.

This interleaving of two kinds of regions finds a natural reflection in the behavior of the drag. As we show below, the dependence of ρ_t on the filling factor ν at a fixed temperature is qualitatively similar to that of σ_{xx} in that it is greatly suppressed deep in the QH phases and peaks at the transitions between them. While we discuss the T dependence of ρ_t everywhere, our results for the compressible regions are particularly interesting. In these we use conjectured forms of $\mathrm{Im}\chi$ to obtain predictions for ρ_t that can be used to test the underlying

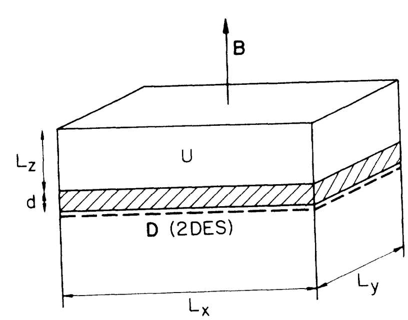

FIG. 1. Schematics of the heterostructure. The shaded area is an insulator, and **B** is the magnetic field.

theories. Assuming critical metallic behavior of the kind discussed by Chalker,[14] we find that $\rho_t \sim T^{2-\eta}$, where η reflects anomalously slow dynamics related to the fractal structure of the critical eigenstates.[14,15] We note that the value of η does not appear to be experimentally observable by other means. In addition to testing the assumptions of Ref. 14, measurements of η at *different* transitions could shed light on the currently debated issue of the number of universality classes in the QH regime.[16] Finally, the FL behavior predicted in Ref. 13 implies that $\rho_t \sim T^2$ as in the $B=0$ experiments.

The paper is organized as follows. First, we discuss a model of a clean quantum wire where the physics of the drag is illustrated transparently in a golden rule calculation; readers interested in the more typical QH systems can skip this without loss of continuity. Next, we discuss two-dimensional systems, particularly the critical region of their disorder dominated transitions. We also discuss the even-denominator compressible states. In most of the following we shall discuss the integer QHE (IQHE), even though the physics is evidently more general; the extension to the fractional QHE (FQHE) is sketched at the end of the paper.

QUANTUM WIRE

We assume that layer D is free of impurities and is subject to a parabolic confining potential, $\frac{1}{2}m\omega_0^2 y^2$. The exactly calculable eigenstates[17] are labeled by the wave vector in the x direction k_d, and the Landau level (LL) index n_d; the corresponding energy levels are $E^{(d)}_{n_d,k_d}$. The electron system in layer U is assumed to be a 3DEG; its eigenstates are labeled by n_u, k_u, and k_z, and the energy levels are $E^{(u)}_{n_u,k_z}$. We calculate ρ_t within the Boltzmann transport approach. The current density j_d flowing in D along the x direction is

$$j_d = \frac{e\hbar}{\pi L_y m^*}\sum_{n_d}\int_{-\infty}^{\infty} dk_d\, g_{n_d k_d} k_d\ , \qquad (2)$$

where $g_{n_d k_d} \equiv -\hbar v_D(\partial f^0_{n_d k_d}/\partial E^{(d)}_{n_d,k_d})k_d$ is the deviation of the electron distribution from the equilibrium Fermi distribution $f^0_{n_d k_d}$, v_D is the drift velocity, and $m^* \equiv m(\omega_c/\omega_0)^2$ in terms of the band mass m and the cyclotron frequency ω_c. The electric field $\mathcal{E}_u$ induced in U is related to the total momentum transfer into this region by

$$\mathcal{E}_u = \frac{\hbar L_x L_z^*}{e\pi^2 N_u}\sum_{n_u}\int_0^{\infty} dk_z \int_{-\infty}^{\infty} dk_u\, k_u \tau^{-1}(k_u,k_z)\ , \qquad (3)$$

where the relaxation rate $\tau^{-1}(k_u,k_z)$ is, to linear order in $g_{n_d k_d}$,

$$\tau^{-1}(k_u,k_z) = \frac{2\pi}{\hbar}\frac{L_z^* L_x^3}{4\pi^4}\sum_{n_u',n_d',n_d}\int_0^{\infty} dk_z' \int_{-\infty}^{\infty} dk_u' \int_{-\infty}^{\infty} dk_d' \int_{-\infty}^{\infty} dk_d |V_{ud,u'd'}|^2$$
$$\times\{P(u,d;u',d') - P(u',d';u,d)\}\delta(E^{(d)}_{n_d,k_d} + E^{(u)}_{n_u,k_z} - E^{(d)}_{n_d',k_d'} - E^{(u)}_{n_u',k_z'})\ ,$$
$$P(u,d;u',d') \equiv f^0_{n_u k_z}(1 - f^0_{n_u',k_z'})\{g_{n_d k_d}(1 - f^0_{n_d',k_d'}) - g_{n_d',k_d'} f^0_{n_d k_d}\}\ ; \qquad (4)$$

here N_u is the number of electrons in U, and $V_{ud,u'd'}$ is the matrix element of the screened Coulomb interaction across the barrier.[18]

We now analyze the variation of $\mathcal{E}_u$ near the transition between successive plateaus, i.e., when either the density or B is varied so that the chemical potential μ crosses the bottom of a LL. The interesting region is when μ lies just above the bottom of the highest occupied LL, where we can ignore scatterings between different Landau levels. For our qualitative purposes it is sufficient to consider the case where the highest occupied levels are the lowest LL in both D and U. The crucial step is to distinguish two different regimes in the position of μ: (a) $k_B T \ll E_F^{(d)}$ and (b) $k_B T \gg E_F^{(d)}$, where $E_F^{(d)} \equiv (\mu - E_{00}^{(d)})$. A bias between the layers is chosen so that in both regimes $E_F^{(u)} \equiv (\mu - E_{00}^{(u)}) \gg k_B T$. Correspondingly, we define $k_F^{(d)} \equiv \sqrt{2m^* E_F^{(d)}}/\hbar$ and $k_F^{(u)} \equiv \sqrt{2mE_F^{(u)}}/\hbar$.

In regime (a), the electron gas in the layer is characterized by a sharp, two point Fermi "surface" $k_d = \pm k_F^{(d)}$. At $T=0$, $\tau^{-1}(k_u,k_z)$ vanishes [cf. Eq. (4), and similar expressions in Refs. 7 and 8]; at small T, it is dominated by forward ($k_d \approx k_d'$) and backward ($k_d - k_d' \approx \pm 2k_F^{(d)}$) scattering processes. The former contribute subdominant orders of T;[19] the latter are $\sim \exp[-(2k_F^{(d)}\ell)^2]\exp[-4k_F^{(d)}d]$, where ℓ is the magnetic length, hence ρ_t is severely suppressed for $E_F^{(d)} > \max\{\hbar^2/2m^*(2\ell)^2, \hbar^2/2m^*(4d)^2\}$. As $E_F^{(d)}$ is reduced below $\min\{\hbar^2/2m^*(2\ell)^2, \hbar^2/2m^*(4d)^2\}$, ρ_t becomes more appreciable and approaches its maximal value as $E_F^{(d)}$ is further reduced, crossing over to regime (b).

In regime (b), where ρ_t is peaked, we assume the hierarchy of energy scales $E_F^{(d)} \ll k_B T \ll E_F^{(u)}$. The 2DES becomes an effectively classical gas, where $k_B T$ replaces $E_F^{(d)}$ as the typical electron energy. Consequently, one should account for the T dependence of q_s, the screening wave vector in D, as well as the effective width of D. The latter is given, roughly, by the effective real-space extent of the electronic states with energy below $E_{00}^{(d)} + k_B T$. We thus obtain the peak transresistivity

$$L_z^* \rho_t^{(0)} \approx \frac{3\pi^5}{16}\frac{\hbar}{e^2}\left(\frac{a_B}{d}\right)^2 \frac{\epsilon^2 (k_F^{(u)})^5 \ell^8 (k_B T)^2}{e^4}\ , \qquad (5)$$

where $a_B \equiv \hbar^2/e^2m$ and ϵ the background dielectric constant. For $m \approx 0.07m_e$ and $\epsilon \sim 10$ (appropriate to GaAs), $d = 200$ Å, $\ell = 100$ Å, $k_F^{(u)} \sim 1/\ell$, and $T = 1$ K, we get $L_z^* \rho_t^{(0)} \sim 10^{-8}\ \Omega\,\mathrm{cm}$.

INVERSION LAYERS

We now turn to realistic systems where, in contrast to the previous example, there are always states at the Fermi level in the bulk. We distinguish two regions: the QH plateaus and the transition regions where metallic behavior is observed.

In the former, the scattering between the layers and hence the drag may be assisted by the localized states, in addition to the states at the edge considered earlier. At low T we expect that these processes are dominated by overlap factors coming from a typical hop, as in the calculation of σ_{xx} in the framework of variable range hopping,[20] and thus contribute $\rho_t \sim e^{-(T_0/T)^{2\alpha}}$ ($\alpha = 1/3, 1/2$ for the cases of Mott hopping and Coulomb gap, respectively). For reasonable system sizes this form holds, except at the very lowest temperatures where the range of the hopping exceeds the sample size, and the edge-edge scattering dominates; for a sample of size $L \sim 10\ \mu$m, we find that the crossover temperature is at most 10^{-4} K.

TRANSITION REGIONS

Here we again distinguish two cases: the disorder dominated critical metals and the clean FL states. In the former the system is in the vicinity of a $T = 0$ critical point with a finite nonzero σ_{xx}. In both cases, ρ_t will be dominated by the extended states that exist in the bulk of the sample. This bulk contribution is conveniently evaluated using the result of Zheng and MacDonald,[10] who showed that to leading nontrivial order in the interaction V between two 2DES's,

$$\rho_t = \frac{\beta\hbar^2}{\pi n^{(u)} n^{(d)} e^2} \int \frac{d^2q}{(2\pi)^2} q^2 |V(q)|^2 \times \int_0^\infty d\omega \frac{\mathrm{Im}\chi_d(q,\omega)\mathrm{Im}\chi_u(q,\omega)}{4\sinh^2(\beta\hbar\omega/2)} , \quad (6)$$

where $\mathrm{Im}\chi_{d(u)}(q,\omega)$ are the dissipative density-density response functions of layer D (U).[21] The leading low T behavior of ρ_t can be obtained by approximating the response functions by their $T = 0$ forms. We note again that as $T \to 0$, the denominator forces the relevant range of ω to vanish, and hence ρ_t becomes sensitive to the form of $\mathrm{Im}\chi_{d(u)}(q,\omega)$ for $\omega \to 0$ *at finite* q.

In applying Eq. (6) to the critical region we are faced with the problem that the relative significance of disorder, interactions, and their interplay is an issue not yet settled. Therefore, we take a dual approach. First, for the well studied noninteracting problem (which appears to be relevant to the experiments thus far[22]), we show that ρ_t vanishes as a known universal power of T. Next, we show that this behavior is generic; at *any* (interacting) critical point ρ_t measures an exponent characteristic of anomalous dissipation in the corresponding universality class.

For noninteracting electrons Chalker and Daniell[14] have shown that the low frequency response at criticality has the form

$$\mathrm{Im}\chi_d(q,\omega) = \frac{(dn^{(d)}/d\mu)\omega\mathcal{D}(q,\omega)q^2}{[\omega^2 + (\mathcal{D}(q,\omega)q^2)^2]} ,$$
$$\mathcal{D}(q,\omega) = D\left(\frac{\mathcal{C}\omega}{q^2}\right)^{\eta/2} . \quad (7)$$

[For $\mathcal{C}\omega > q^2$, $\mathcal{D}(q,\omega) \approx D = 0.087/\hbar(dn^{(d)}/d\mu)$; $\mathcal{C} \approx 60\hbar(dn^{(d)}/d\mu)$ where $dn^{(d)}/d\mu$ is the density of states at the band center, and the universal exponent $\eta \approx 0.38$.[23]] On substituting this in Eq. (6) we obtain, for two identical layers

$$\rho_t \approx 3.1 \times 10^{-4} \frac{h}{e^2} \left(\frac{k_B T}{\hbar D n^{(d)}}\right)^2 \left(\frac{\epsilon}{e^2 d(dn^{(d)}/d\mu)}\right)^2 \times \left(\frac{\mathcal{C} d^2 k_B T}{\hbar}\right)^{-\eta} . \quad (8)$$

Hence, in this scenario, ρ_t in the critical region is *parametrically* enhanced at low T over noncritical filling factors, and its T dependence directly measures η (with a nonuniversal prefactor). As noted by Chalker,[14] the nonzero value of η reflects the presence of large amplitude fluctuations in the multifractal, critical eigenstates. Also, as shown in Ref. 15, these lead to anomalously slow relaxation of the local density fluctuations. Hence, the enhancement of ρ_t is entirely in accord with our earlier discussion of the physics. For a sample of mobility $\sim 10^6\ \mathrm{cm^2/V\,s}$, $n^{(d)} \sim 10^{11}\ \mathrm{cm}^{-2}$, $d = 200$ Å, and $B = 10$ T, we estimate $\rho_t \approx 10\ \mathrm{m}\Omega$ at $T \approx 0.1$ K. As with the scaling of σ_{xx}, the behavior of ρ_t becomes noncritical at a crossover temperature $T^* \sim 1/\xi$.[22]

We now consider the situation at an interacting critical point, with unknown exponents. As $\mathrm{Im}\chi$ is the correlator of a conserved density it has, by standard renormalization group arguments, the critical scaling form

$$\mathrm{Im}\chi(q,\omega) = q^{d-z} f(\omega/q^z), \quad (9)$$

where z is the dynamic scaling exponent. $\mathrm{Im}\chi$ is odd in ω, and away from criticality we expect $\mathrm{Im}\chi \sim \omega f(q)$ at small ω for fixed q. Assuming that the critical fluctuations modify this linear dependence we parametrize the deviation by postulating $f(x) \sim x^{1-\eta/2}$ for $x \ll 1$.[24] For our problem $\mathrm{Im}\chi \sim \omega^{1-\eta/2} q^{2-2z+\eta z/2}$ for $\omega \ll q$; this agrees with the noninteracting form with $z = 2$. Substituting this in Eq. (6) gives $\rho_t \sim T^{2-\eta}$ at low T. Hence an enhanced drag at criticality would still measure an exponent characteristic of anomalous low frequency dissipation.[25]

In extremely clean samples the transition regions are believed to contain FL states.[13] It is not known whether, at the longest length scales, these scale to a new metallic fixed point or to the disorder dominated one considered earlier. Regardless, for realistic sample sizes we can use the form $\mathrm{Im}\chi \sim \omega/q$ (Ref. 13) to predict that $\rho_t \sim T^2$ as in the $B = 0$ case. This repetition of the zero field behavior at a large field would be striking evidence for the existence of the FL states.

Finally, some comments on the FQHE. The qualitative variation of ρ_t with filling factor at fixed T derives from the incompressible-compressible structure which is the same in the IQHE and the FQHE. The behavior of the quantum wire should be similar; although there are backscattering processes sensitive to the Luttinger liquids at the edges we note that ρ_t is significant only when $k_BT \gg E_F$. For disordered samples, the variable range hopping form should hold deep in the plateaus, where the fractional statistics of the quasiparticles is likely unimportant. As for the critical region, even less is known than for the IQHE; hence, as already emphasized, we expect measurements of ρ_t, and hence of η, to be particularly illuminating.

ACKNOWLEDGMENTS

We are very grateful to A. H. MacDonald for much useful advice and for informing us of his work. We thank G. Baym, J. Eisenstein, J. Engelbrecht, E. Fradkin, S. Girvin, A. Jauho, B. Laikhtman, A. Leggett, and B. Shklovskii for helpful discussions, and A. H. MacDonald and S. Kivelson for comments on the manuscript. This work was supported in part by the Beckman Foundation (E.S.), and NSF Grants Nos. DMR 91-22385 and DMR 91-57018, and the Aspen Center for Physics (S.L.S.).

[1] See, e.g., *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).

[2] For "strong coupling" physics *not* considered in this paper see, for example, A. H. McDonald, Surf. Sci. **229**, 1 (1990); T. Fukuzawa, E. E. Mendez, and J. M. Hong, Phys. Rev. Lett. **64**, 3066 (1990); L. Swierkowski, D. Neilson, and J. Szymanski, *ibid.* **67**, 240 (1991); Y. W. Suen, L. W. Engel, M. B. Santos, M. Shayegan, and D. C. Tsui, *ibid.* **68**, 1379 (1992); J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, K. W. West, and S. He, *ibid.* **68**, 1383 (1992).

[3] M. B. Pogrebinskii, Fiz. Tekh. Poluprovodn. **11**, 637 (1977) [Sov. Phys. Semicond. **11**, 372 (1977)]; P. M. Price, Physica (Amsterdam) **117B**, 750 (1983).

[4] P. M. Solomon, P. J. Price, D. J. Frank, and D. C. La Tulipe, Phys. Rev. Lett. **63**, 2508 (1989).

[5] T. J. Gramila, J. P. Eisenstein, A. H. MacDonald, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **66**, 1216 (1991).

[6] U. Sivan, P. M. Solomon, and H. Shtrikman, Phys. Rev. Lett. **68**, 1196 (1992).

[7] B. Laikhtman and P. M. Solomon, Phys. Rev. B **41**, 9921 (1990); Superlatt. Microstruct. **10**, 89 (1991).

[8] A. P. Jauho and H. Smith, Phys. Rev. B **47**, 4420 (1993).

[9] H. C. Tso and P. Vasilopolous, Phys. Rev. B **45**, 1333 (1992); H. C. Tso, P. Vasilopolous, and F. M. Peeters, Phys. Rev. Lett. **68**, 2516 (1992); I. I. Boiko, P. Vasilopolous, and Yu. Sirenko, Phys. Rev. B **45**, 13 538 (1992).

[10] L. Zheng and A. H. MacDonald, Phys. Rev. B **48**, 8203 (1993). The extension to $B \neq 0$ is discussed in L. Zheng, Ph.D. thesis, Indiana University, 1993.

[11] The dc conductivity is defined by the order of limits $\lim_{\omega\to 0}\lim_{q\to 0}\omega/q^2\mathrm{Im}\chi(q,\omega)$.

[12] H. Levine, S. B. Libby, and A. M. M. Pruisken, Phys. Rev. Lett. **53**, 1915 (1983); D. E. Khmelnitiskii, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1983)].

[13] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993); V. Kalmeyer and S.-C. Zhang, *ibid.* **46**, 9889 (1992).

[14] J. T. Chalker, J. Phys. C **20**, L493 (1987); J. T. Chalker and G. J. Daniell, Phys. Rev. Lett. **61**, 593 (1988); J. T. Chalker, Physica A **167**, 253 (1990).

[15] B. Huckestein and L. Schweitzer, Phys. Rev. Lett. **72**, 713 (1994).

[16] W. Chen, M. P. A. Fisher, and Y. S. Wu, Phys. Rev. B **48**, 13 479 (1993).

[17] D. Childers and P. Pincus, Phys. Rev. **177**, 1036 (1969).

[18] L_z^* is the effective thickness of layer U; for $L_z < \lambda_{sc}$ (the screening length in U), $L_z^* \approx L_z$, and for $L_z > \lambda_{sc}$, $L_z^* \approx \lambda_{sc}$.

[19] E. Shimshoni and S. L. Sondhi (unpublished).

[20] D. G. Polyakov and B. I. Shklovskii, Phys. Rev. Lett. **70**, 3796 (1993).

[21] The reader may be concerned about our using a weak coupling result near a critical point. Our belief is that weak interlayer coupling is, at least practically, irrelevant at the QH critical points. Experiments, such as that by Eisenstein *et al.* (Ref. 2), that have tracked the collapse of correlated interlayer states with separation, indicate that such a regime is indeed accessible. Also, to treat the long range of the interaction, we follow Ref. 10 in replacing the interaction and the response functions by their screened counterparts.

[22] The observed scaling [H. P. Wei, D. C. Tsui, M. Paalanen, and A. M. M. Pruisken, Phys. Rev. Lett. **61**, 1294 (1988)] in the transition regions is consistent with the localization length exponent calculated from studies of noninteracting electrons [see Y. Huo and R. N. Bhatt, Phys. Rev. Lett. **68**, 1375 (1992), and references therein]. The simplest assumption is, therefore, that for "Fermi liquid" reasons the screened response functions are given correctly by the noninteracting fixed point; for a different interpretation see D. H. Lee, Z. Q. Wang, and S. A. Kivelson, *ibid.* **70**, 4130 (1993).

[23] The exponent η is related to the generalized dimension D_2 of the critical states by $D_2 = 2 - \eta$ (Refs. 14 and 15).

[24] For $x \gg 1$, $f(x) \sim x^{(d-2-z)/z}$ consistent with $\sigma(\omega) \sim \omega^{(d-2)/z}$; see, for example, M. P. A. Fisher, G. Grinstein, and S. M. Girvin, Phys. Rev. Lett. **64**, 587 (1990).

[25] Possibly also at other two-dimensional phase transitions.