RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 51, NUMBER 14 1 APRIL 1995-II

Role of vortices in the mutual coupling of superconducting and normal-metal films

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(Received 31 January 1995)

I propose a possible explanation to a recently observed "cross-talk" effect in metal-insulator-metal trilayers, indicating a sharp peak near a superconducting transition in one of the metal films. Coulomb interactions are excluded as a dominant coupling mechanism, and an alternative is suggested, based on the local fluctuating electric field induced by mobile vortices in the superconducting layer. This scenario is compatible with the magnitude of the peak signal and its shape; most importantly, it addresses the *nonreciprocity* of the effect in exchanging the roles of the films.

In a recent experiment[1] Giordano and Monnier have observed an intriguing effect in structures composed of two parallel metal films, separated by a thick insulating layer which prohibits tunneling. The voltage measured on one metal film in response to a transport current in the other exhibited a peak in the narrow interval of temperatures, $T_c<T<T_{\rm MF}$, in which one of the films (a dirty Al) undergoes a superconducting–to–normal-metal transition (SNT), while the other (typically Sb) is a normal metal (these films are denoted below by S and N, respectively). Out of this interval, the induced voltage was negligibly small. Most astonishingly, the effect was found to be nonreciprocal: the induced voltage for given T and drive current depended on which film carries the current. In particular, in the case where current is driven in S ("case A"), the voltage was a *nonlinear* function of the current, and negative (i.e., opposite in sign to the voltage generating the current); in "case B," where the current is driven in N, the voltage is approximately a linear function of the current, and positive. The effect was qualitatively the same in the presence of externally applied magnetic fields (which primarily modify T_c and $T_{\rm MF}$)—the signal was restricted to the immediate vicinity of the SNT in S.

The purpose of the present work is to propose a coupling mechanism between superconducting and normal-metal films, which provides a plausible explanation to most of the observations described above. At first sight, the effect is reminiscent of the Coulomb drag observed in semiconductor heterostructures,[2–4] which results from Coulomb interactions between charge carriers across the insulating barrier separating two conducting layers. As was pointed out in Ref. 1, in the present case where *metals* are involved Coulomb drag is far too weak to explain their data at the peak of the signal.[5] As I show below, a certain enhancement of the drag may be generated by the presence of vortices in S near the SNT; however, for the parameters of the system at hand, this enhancement is insignificant, and I conclude that Coulomb interactions should be ruled out as a dominant coupling mechanism. I suggest an alternative, which can be viewed as a coupling between charge carriers in N and vortices in S, mediated by the flux tubes carried by the latter. The fact that this mechanism (named "inductive coupling") is dominated by the dynamics of excitations confined to only one of the layers (i.e., vortices), turn out to be a key to understanding the nonreciprocity of the effect.

A significant clue is the effect being restricted to the region of SNT in S, and maximized at an intermediate temperature T_p, in which the resistance of that layer is finite but significantly smaller than the normal-state value. It is therefore natural to suspect (as was also remarked in Ref. 1) that the presence of vortices in S is playing a crucial role. Vortices exist in these thin, dirty Al films since the effective penetration depth for perpendicular magnetic fields, Λ, is typically much larger than the coherence length $\xi_{\rm eff}$:[6]

$$\Lambda=\frac{2\lambda_L^2}{d}\left(\frac{\xi_0}{\ell}\right) \quad , \quad \xi_{\rm eff}=(\xi_0\ell)^{1/2} \quad , \tag{1}$$

where d is the film thickness, ℓ is the elastic mean free path, and λ_L, ξ_0 are, respectively, the intrinsic penetration depth and the $T=0$ coherence length of the material. Free vortices can be thermally excited near the SNT, even in the absence of externally applied magnetic field, and the phase slips associated with their motion are known to be a major source of the finite resistance near the transition.[7,8]

The strength of coupling between the metallic layers is measured by the transresistivity

$$\rho_{ns}=\mathscr{E}_n/j_s\,, \tag{2}$$

where $\mathscr{E}_n$ is the parallel electric field induced in N in response to a current density j_s in S (adapting the scenario of "case A"). It is most useful to relate this transport coefficient (in linear response) to a correlation function, similarly to the Kubo formula for conductivities (which, by itself, is less convenient in the problem at hand). Employing the memory functional formalism,[9,4] one can indeed express the dc transresistivity (at finite T and to lowest order in the interlayer interaction) as

$$\rho_{ns}=\frac{1}{k_BTn_nn_se^2A}\int_0^\infty dt\langle F_n(t)F_s(0)\rangle\ ; \tag{3}$$

here n_i is the electron density in the (two-dimensional) layer i, A is the cross section area of the layers, and F_i is the time-dependent, zero-wave-vector Fourier component of the force density operator, acting on charge-carriers in layer i.

Note the resemblance to the Kubo formula, with the current operators J_i being replaced by their conjugates $\dot{J}_i \propto F_i$. Searching for the dominant interlayer coupling mechanism amounts to identifying the components of F_i dominating the above correlator.

In the rest of the paper, I first consider the Coulomb drag effect in the vicinity of the SNT, showing that the presence of vortices may enhance it. This part of the work is concluded by excluding its relevance to the experiment in Ref. 1; however, it is an interesting possibility that may be detectable in different experimental scenarios (e.g., involving high-T_c superconducting films), and to the best of my knowledge has not been proposed elsewhere. In the last and main part of this work I construct the inductive coupling picture, show that it is compatible with most aspects of the experiment, and elaborate on the difficulties, possible resolutions, and suggestions for further research.

Enhancement of Coulomb drag. "Coulomb drag" denotes the finite transresistivity resulting from Coulomb interactions between charge fluctuations at the different layers. Pictorially, moving charges in one layer exert a force on charges in the other, thus "dragging" them along the direction of the drive current flow. The strength of this interlayer coupling indicates the ability of electronic states in the layers to support inhomogeneities in the charge density, which are necessary to establish forces between the layers. Substituting the Coulomb force for F_i in Eq. (3) yields[4]

$$\rho_{ns}=\frac{\hbar^2}{k_BT\pi n_n n_s e^2}\int\frac{d^2q}{(2\pi)^2}\,q^2|V(q)|^2 \times\int_0^\infty d\omega\,\frac{\mathrm{Im}\chi_n(q,\omega)\mathrm{Im}\chi_s(q,\omega)}{4\sinh^2(\hbar\omega/2k_BT)}\quad, \tag{4}$$

where V is the screened interlayer Coulomb interaction, and $\mathrm{Im}\chi_i(q,\omega)$ is the dissipative part of the density-density response function of layer i. The physics described above is reflected by the $T\to 0$ behavior of ρ_{ns} being sensitive to the $\omega\to 0$, *finite* q form of $\mathrm{Im}\chi_i(q,\omega)$.

Far below the SNT in layer S, frictional drag is expected to vanish on the same basis as any dissipation channel, due to the gap to excitations deep in the superconducting state. Above the transition, $\rho_{ns}\sim 10^{-6}\ \Omega$,[1–4] which is negligibly small. However, following the preceding discussion, the Coulomb drag could in principle be enhanced in a scenario where density fluctuations of finite q are favored. I argue that such a scenario is realized in the close vicinity of the SNT (just below T_{MF}), where a small superconducting gap is opened. Due to the formation of vortices, this gap is not uniform—it vanishes in the core of the vortices, and grows towards their periphery over a length scale ξ_{eff}. Hence, the normal quasiparticle excitations contributing to $\mathrm{Im}\chi_s^{(-)}$ (the response function for $T<T_{\mathrm{MF}}$) accumulate at the vortex cores.

I simulate this situation by a simple ansatz for the quasiparticle eigenstates. Their basis wave functions, $\psi_i^-(\mathbf{r})$, are related to the normal-state, $T>T_{\mathrm{MF}}$ eigenstates $\psi_i^+(\mathbf{r})$ via an envelope function, which mimics the spatial variation of the gap in the presence of a dense array of vortices:

$$\psi_i^-(\mathbf{r})=N\psi_i^+(\mathbf{r})[1+(l_v/\xi_{\mathrm{eff}})\cos(\mathbf{k}\cdot\mathbf{r})]\quad, \tag{5}$$

where l_v is the typical distance between vortices, $|\mathbf{k}|=1/l_v$, and $N\equiv[1+(l_v/\xi_{\mathrm{eff}})^2]^{-1/2}$.[10] Consequently, $\chi_s^{(-)}(\mathbf{q},\omega)$ (Ref. 11) can be cast in a form involving $\chi_s^{(+)}$ (of $T>T_{\mathrm{MF}}$) with $\mathbf{q}$ shifted by $\pm\mathbf{k}$. When substituted in Eq. (4), these terms involving $\chi_s^{(+)}(\mathbf{q}\pm\mathbf{k},\omega)$ will dominate, due to the pole established at $\omega=0$, $\mathbf{q}=\pm\mathbf{k}$. Assuming further that the normal state (of both metallic layers, S and N) is in the diffusive regime, so that $\chi_s^{(+)}(\mathbf{q},\omega)=(dn/d\mu)Dq^2/(Dq^2-i\omega)$ (with $dn/d\mu$ the density of states and D the diffusion coefficient), I obtain an approximate expression for $\rho_{ns}^{(-)}$,

$$\rho_{ns}^{(-)}\sim\rho_{ns}^{(+)}(\hbar D/l_v^2k_BT)^{1/2}(l_v/\xi_{\mathrm{eff}})^4\quad. \tag{6}$$

The first factor in parentheses tends to *enhance* the drag, in comparison with its value for $T>T_{\mathrm{MF}}$, reflecting the intuition that nonuniform density fluctuations are stabilized by the nonuniform gap. However, this competes with the second, suppressing factor, associated with the amplitude of the gap modulation. Near the SNT, the former increases with T (with increasing vortex density), while the latter decreases due to the divergence of $\xi_{\mathrm{eff}}(T)$. On the face of it, this behavior is qualitatively compatible with the observation in Ref. 1. However, the maximum of $\rho_{ns}^{(-)}$ is achieved at $l_v\sim\xi_{\mathrm{eff}}$, in which case it is enhanced with respect to $\rho_{ns}^{(+)}$ by at most an order of magnitude, for the parameters of the system at hand. As $\rho_{ns}^{(+)}$ is extremely small in the first place, one must conclude that this enhancement of Coulomb drag is irrelevant to the present experiment. A more pronounced enhancement may be expected, however, if the Al is replaced by a material with higher T_c, shorter ξ_{eff}, and lower D (e.g., near a superconducting-insulator transition), and could serve as an interesting demonstration of coupling between density fluctuations and phase fluctuations of the superconducting order parameter.

Inductive coupling. An alternative coupling mechanism between the layers is associated with the motion of free vortex and antivortex excitations in S at $T_c<T<T_{\mathrm{MF}}$. These excitations are accompanied by (self-consistently generated) magnetic flux tubes, which extend out perpendicularly to S and penetrate the neighboring layer N. Note that the magnetic field of a vortex varies over length scale $\Lambda\sim 25\ \mu$m [for $\lambda=500$ Å, $\xi_0=1.6\ \mu$m, $\ell\sim 100$ Å, and $d\sim 350$ Å in Eq. (1)], that is much larger than the thickness of the trilayer device, so that bending of the field lines out of S can be neglected. In the absence of an external magnetic field, the fluctuating magnetic field thus generated in N, $\mathbf{B}(\mathbf{r},t)$, averages to zero over the sample area, as vortex and antivortex excitations are equally likely. However, the local, instantaneous time dependence of $\mathbf{B}(\mathbf{r},t)$ induces a fluctuating electric field $\mathbf{E}_n(\mathbf{r},t)$. Since $\mathbf{E}_n(\mathbf{r},t)$ is correlated with the electric field in S, a finite interlayer coupling coefficient is established.

I first focus on "case A," in which the drive current is passed in the layer S. To evaluate the transresistivity ρ_{ns} using Eq. (3), I consider the force fluctuation $\mathbf{F}_s(\mathbf{r},t)$ acting on the charge carriers in S due to the phase slips associated with vortex motion:[7]

$$\mathbf{F}_s(\mathbf{r},t)=(\phi_0/c)\hat{\mathbf{z}}\times\mathbf{J}_v(\mathbf{r},t)\quad, \tag{7}$$

where $\phi_0 = hc/2e$ is the flux quantum, $\hat{\mathbf{z}}$ is a unit vector perpendicular to the layer, and $\mathbf{J}_v(\mathbf{r},t) = n_v(\mathbf{r},t)\mathbf{v}_v(\mathbf{r},t)$ is the fluctuating *vortex* current density [$n_v(\mathbf{r},t)$ and $\mathbf{v}_v(\mathbf{r},t)$ being the vortex density and velocity, respectively]. The force in N is, in turn, $\mathbf{F}_n(\mathbf{r},t) = e\mathbf{E}_n(\mathbf{r},t)$, where $\mathbf{E}_n$ satisfies

$$\nabla \times \mathbf{E}_n(\mathbf{r},t) = -(1/c)\partial \mathbf{B}(\mathbf{r},t)/\partial t \ . \tag{8}$$

$\mathbf{B}(\mathbf{r},t)$ is assumed the form

$$\mathbf{B}(\mathbf{r},t) = \phi n_v(\mathbf{r},t)\hat{\mathbf{z}}, \tag{9}$$

where ϕ is an effective flux transferred by a single moving vortex. The distance that a vortex can traverse freely is limited by l_v, the typical intervortex spacing (beyond which it is likely to be annihilated by an antivortex). In the present case, near the center of the SNT region $l_v \ll \Lambda$, and hence the flux transfer generated in N along with a phase slip of 2π in S *is much smaller than* ϕ_0. Approximating the magnetic field within a radius Λ of a vortex by its average, and using a crude ratio-of-areas argument, I find

$$\phi = f\phi_0 \ , \quad f \sim l_v/2\Lambda \ . \tag{10}$$

Equation (8) combined with Eqs. (9), (10) yields [ignoring fast fluctuations with $\nabla \cdot v_v \neq 0$ (Ref. 7)]

$$\mathbf{F}_n(\mathbf{r},t) = (\phi/c)\hat{\mathbf{z}} \times \mathbf{J}_v(\mathbf{r},t) \ . \tag{11}$$

Comparing to Eq. (7), one observes that $\mathbf{F}_n$ differs from $\mathbf{F}_s$ only by the reduction factor f relating ϕ to ϕ_0, reflecting the fact that it is induced by the "magnetic fraction" of the very same vortices. Using Eq. (3), I thus obtain

$$\rho_{ns} = \frac{1}{k_B T}\left(\frac{\phi\phi_0}{c^2}\right)\int_0^\infty dt \langle J_v(t) J_v(0)\rangle = \left(\frac{\phi\phi_0}{c^2}\right)\sigma_v \tag{12}$$

[$J_v(t)$ is the $q=0$ Fourier component of $\mathbf{J}_v(\mathbf{r},t)$, along the direction of the drive current]; the last equality follows from a "Kubo formula" for the vortex conductivity σ_v. I next assume that the resistivity of S near T_p is dominated by the vortex flow, i.e., $\rho_s = (\phi_0/c)^2\sigma_v$.[7] This implies that, within the assumptions above,

$$\rho_{ns}(T) = f(T)\rho_s(T) \ . \tag{13}$$

Since $\rho_s(T)$ increases as a function of T, while $f(T)$ decreases [see Eq. (10), noting that l_v decreases and, at the same time, Λ diverges], $\rho_{ns}(T)$ is nonmonotonic for $T_c < T < T_{\rm MF}$, and vanishes for $T < T_c$ (where $\rho_s = 0$) and $T > T_{\rm MF}$ (where $f = 0$). This is in qualitative agreement with the experimental result.

So far I relied on the assumption of *linear response*, which is in fact inconsistent with the experiment. Before discussing the consequence of relaxing this assumption, note that within linear response, the Onsager relations would imply reciprocity, namely $\rho_{ns} = \rho_{sn}$ (where ρ_{sn} is the transport coefficient compatible with "case B"). To confirm this, I focus on "case B," in which a current density $\mathbf{j}_n$ is passed in N. This current, enforced by the external driving source, applies a Lorentz force on the flux tubes carried by vortices in S,[8] and thus on the vortices themselves:

$$\mathbf{F}_L = (\phi/c)\hat{\mathbf{z}} \times \mathbf{j}_n \ , \tag{14}$$

where ϕ is given by Eq. (10). $\mathbf{F}_L$ resembles the force on a vortex in the presence of a supercurrent,[7] except that the "vortex charge" is effectively renormalized by the fraction f. An equivalent scenario, in terms of the impact on the vortices, could be achieved by driving a current $\mathbf{j}_s = f\mathbf{j}_n$ *directly through S*. In view of this equivalence, I find, as expected,

$$\rho_{sn} = f\rho_s = \rho_{ns} \ . \tag{15}$$

The violation of reciprocity observed in Ref. 1 can indeed be associated with *nonlinearity*, and the argument is as follows: in "case A," the applied supercurrents j are assumed sufficiently strong to push the vortices into the nonlinear response regime. The principal implication on Eq. (12) is that the vortex response is replaced by a current-dependent coefficient, $\sigma_v(j)$. However, in "case B" where *the same* current j is supplied to N, the vortices respond as if a much smaller current fj is driven directly in S; their conductivity is hence well approximated by $\sigma_v(0)$. This distinction between the two cases is clearly consistent with the experiment.

The estimated magnitude of ρ_{sn} at the peak T_p is also in accord with the data presented in Ref. 1. I evaluate f using the Landau-Ginzburg expression $\Lambda(T_p) = \Lambda(0)[T_{\rm MF}/(T_p - T_{\rm MF})]$ [see Eq. (1)], and the analysis of Ref. 7 to estimate $l_v(T_p)$. For the experimental values $T_c = 1.77$ K, $T_p = 1.81$ K, and $T_{\rm MF} = 1.86$ K, I get $f \sim 10^{-4}$, which implies a peak voltage $V_s \sim 200$ nV in S [with film resistance $R_s(T_p) \sim 400\ \Omega$] for a current $I = 7\ \mu$A driven in N. This is roughly a factor of 2.5 *larger* than the experimental value. A better quantitative agreement cannot be expected within the crude assumptions involved in this work. In particular, my overestimate of the effect may be due to overestimating the contribution of vortex dynamics to R_s, disregarding other degrees of freedom.

Finally, the sign of the effect in the linear case (B) can be reconciled with the mechanism proposed here: the arguments leading to Eq. (15) also imply that it is identical to the sign of a voltage established in S in response to an "equivalent," in-layer supercurrent. The negative sign in case A is the one aspect of the experiment still open for interpretation. It is not, however, in contradiction with the picture constructed so far: since it occurs in the nonlinear response regime,[13] there is a good reason to suspect that it involves processes not included in the present scheme (which, e.g., introduce a force on the vortices in the opposite direction). Most likely are thermal conduction processes, leading to flow dictated by a temperature gradient rather than electric current.[12] A more elaborate theory is required to clear this point. Further experimental work could also shed some light, e.g., a more detailed study of the dependence on drive current. It should be noted that the inductive coupling mechanism can be distinguished from Coulomb drag by a multitude of tests: the latter is more sensitive to the distance between layers and to the sign of charge carriers; moreover, for a narrower strip[14] Coulomb interaction is expected to be enhanced, while induction is further limited by the width of the strip. I conclude by pointing out that in view of the interpretation proposed in this paper, the "cross-talk" effect is a suggestive probing technique for the dynamical properties of vortices (in distinction from other degrees of freedom in a superconducting film).

I am grateful to N. Giordano for showing me his data prior to publication, and to him as well as S. Girvin, A. Leggett, and S. Sondhi for illuminating discussions. This work was supported by the Beckman Foundation.

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