

ZENER TRANSITIONS FOR A SPIKY DRIVE

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We investigate the problem of Zener transitions in a system where the external drive is spiky. This describes for example a Josephson junction driven by a sequence of current pulses. In the limit of small spikes we recover the Landau-Zener expression. In the limit where the discreteness of the spikes is more pronounced the transition probability approaches unity. The crossover is characterized by strong interference effects.

1. INTRODUCTION

Charging effects in ultra-small Josephson junctions and Josephson arrays have been studied extensively in the recent years (1). The dynamics of such systems is determined by the interplay between the Zener dynamics in the band structure and the dissipation. Much of the theoretical work employed models which are based on the assumption of an ideal current source. However, it became clear that this is difficult to achieve in an experiment. One possibility to realize the large resistance necessary to produce a current source is to shunt the junction in series with a network of other junctions (2). However, this introduces a new problem: the conductance in the array involves a rapid discharging due to Cooper pair tunneling or stochastic single electron tunneling. After each charge step in one junction the charge on all islands of the array is redistributed (on a rapid time scale). As a result the charge $Q(t)$ injected into the junction of interest is a sequence of random steps (see Fig. 1).

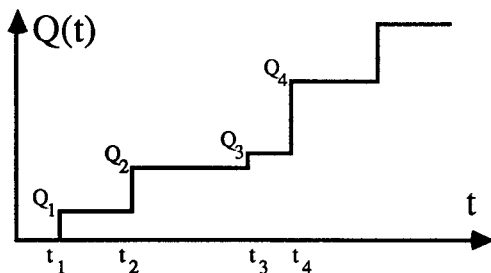


Figure 1: Time evolution of the charge $Q(t)$

We study the effect of such a non-ideal current source by considering a model where a single Josephson junction is driven by a sequence of sharp current pulses. We calculate the probability of transitions from the lower energy level to upper energy levels in the vicinity of the first narrow gap of the band structure. We note that the transition from the lower energy

level to the upper energy levels leads to a destruction of the Bloch oscillations and to a crossover to a normal behavior of the junction (1). We ignore finite temperature and phase breaking effects. For a given average current, the transition probability approaches the Landau-Zener expression, which applies to a constant current, as the size of the charge steps is reduced. In the opposite limit the transition probability is close to unity. In section 2 we describe the model, in section 3 we present the results.

2. THE MODEL

In the vicinity of the first narrow gap, the junction behaves effectively as a two-level system with the Hamiltonian

$$H[Q] = 4 E_C q \sigma_z + \frac{1}{2} E_J \sigma_x \quad \text{where } q \equiv (Q-e)/2e. \quad [1]$$

Here σ_z, σ_x are Pauli matrices, $E_C = e^2/2C$ is the scale of the charging energy and E_J is the Josephson energy ($E_J \ll E_C$). We treat Q as a time-dependent parameter. The instantaneous (adiabatic) energy levels, which correspond to the adiabatic states $|Q(t), \pm\rangle$, are

$$\epsilon_{\pm}[Q(t)] = \pm 4 E_C \sqrt{q^2(t) + g^2} \quad [2]$$

where $g \equiv E_J/(8E_C)$. In the case of an ideal current source, $Q(t) = I_x t$. If the initial state of the system is $|Q(-\infty), -\rangle$, the transition probability to the upper adiabatic states is given by the Landau-Zener expression (3)

$$P_z \equiv |\langle Q(+\infty), + | \psi(t \rightarrow \infty) \rangle|^2 = e^{-\frac{\pi}{2} \gamma}, \quad \gamma \equiv \frac{e E_J^2}{\hbar 4 E_C I_x} \quad [3]$$

Note that the Zener probability is exponentially small for $\gamma > 1$ (in the adiabatic regime). This result is recovered by our model, provided that the step size is sufficiently small.

In the case where $Q(t)$ is described by fig. 1, in each time-interval $t_k < t < t_{k+1}$ the charge is constant, $Q(t) = Q_k$. The state of the system can be decomposed as

$$\psi(t) = C_- (Q_k) |Q_k, -\rangle e^{-i\epsilon_-(Q_k)t/\hbar} + C_+ |Q_k, +\rangle e^{-i\epsilon_+(Q_k)t/\hbar} \quad [4]$$

Using the continuity of $\psi(t)$, we find a relation between the coefficients of $\psi(t)$ in consecutive steps:

$$C_{\pm}(Q_{k+1}) = \langle Q_{k+1, \pm} | Q_{k, \rightarrow} \rangle e^{i[\epsilon_{\pm}(Q_{k+1}) - \epsilon_{\pm}(Q_k)]t_k/\hbar} C_{\pm}(Q_k) + \langle Q_{k+1, \pm} | Q_{k, +} \rangle e^{i[\epsilon_{\pm}(Q_{k+1}) - \epsilon_{\pm}(Q_k)]t_k/\hbar} C_{\pm}(Q_k) \quad [5]$$

where

$$\langle Q_{k+1, s} | Q_{k, s'} \rangle = \frac{g^2 + f_{k+1, s} f_{k, s'}}{(g^2 + f_{k+1, s}^2)^{1/2} (g^2 + f_{k, s'}^2)^{1/2}}$$

Here $f_{k, s} = q_k + s(q_k + g^2)^{1/2}$, and $s = \pm 1$. Assuming that the system was prepared at $t = t_1$ at the lower adiabatic level (i.e., $C_{-}(Q_1) = 1$, $C_{+}(Q_1) = 0$), the probability to perform a transition to the upper level after one step is $| \langle Q_2, + | Q_1, - \rangle |^2$. In particular for $q_1 < 0$ and $q_2 > 0$, with $|q_1|, q_2 \gg g$ (i.e., distant from the transition region), the transition probability is close to unity. If, however, the step size is decreased so that N intermediate steps are included between the initial charge q_1 and the final charge q_N , the final transition amplitude is determined by a coherent summation over 2^{N-1} terms. Each term describes a partial wave, which "spends" part of the time in the lower level and the other part in the upper. The intricate structure of the interfering partial waves, eventually gives rise to the exponential Zener transition probability, eq. [3], in the limit where the step-size approaches zero. Formally, employing eq. [5] in the limit $\delta q \equiv (q_{k+1} - q_k) \rightarrow 0$, one indeed recovers the differential expression for the transition amplitude in the adiabatic limit.

3. RESULTS

We have calculated numerically the probability of a transition to the upper level after N steps $P_t = |C_{+}(Q_N)|^2$, setting the initial conditions $C_{+}(Q_1) = 0$, $C_{-}(Q_1) = 1$. The initial and final charges were chosen so that $q_1 \ll -g$ and $q_N \gg g$. We assumed that the step size and the step duration are equal for all $1 < k < N$. To simulate the realistic situation with many transition attempts and different initial conditions, we average over q_1 with a uniform distribution within one step-size.

In fig. 2 the transition probability P_t is plotted as a function of the step size δq , for two different values of γ . The parameter γ is defined in eq. [3], where I_x denotes the average current. The figure demonstrates the reduction of P_t from a value close to unity to the Landau-Zener exponential expression. This reduction is not smooth and the fluctuations reflects the effect of quantum interference. They will be smoothed if the steps are chosen less regular. Note also that for smaller gap g (which corresponds to smaller γ in the figures), the reduction of P_t from unity appears for a smaller step-size. The reason is that effectively the steps which contribute to the delicate interference, and thus to the exponential reduction, are those that accumulate in the transition region ($q \sim g$). In fig. 3 we show the current dependence of P_t for fixed g . One can see that in the region of small currents the Landau-Zener expression is recovered for smaller δq .

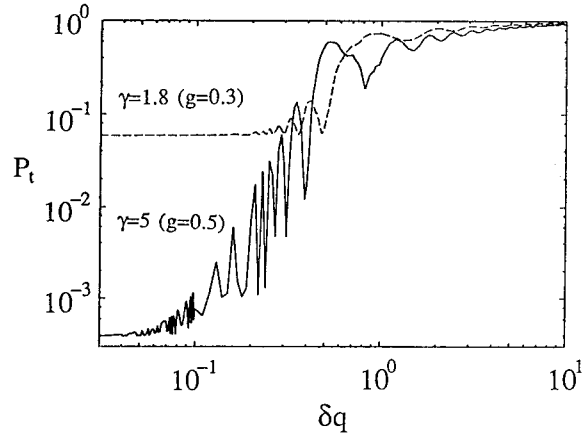


Figure 2: The transition probability P_t versus step size for two values of g . The Landau-Zener limit is recovered for small δq .

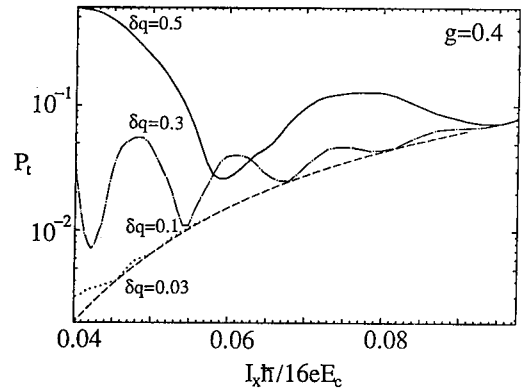


Figure 3: P_t as a function of the mean current for different step sizes δq . For the smallest δq the Landau-Zener limit is recovered in the range of currents shown.

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