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Localization of the Hall resistivity at high magnetic fields: absence of the quantized Hall insulator

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Abstract

We present a real-space renormalization-group study of localization in a two-dimensional hierarchical network model that is designed to represent *phase-coherent* electron transport in the quantum-Hall regime. Scaling expressions for both the longitudinal and Hall resistivities are derived. In agreement with recent numerical studies, we find that the Hall resistivity is quantized in the metallic phase but *diverges* in the insulating phase. This suggests that the characteristics of a *quantized Hall insulator* can occur only in the presence of a strong dephasing mechanism. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The low-temperature transport properties of disordered two-dimensional (2D) electron systems subject to a strong perpendicular magnetic field B exhibit a multitude of transitions between distinct phases. See Ref. [1]. These include primarily the various quantum Hall (QH) phases, characterized by quantized plateaus in the Hall resistivity at the values $\rho_H = h/e^2\nu$, accompanied by a vanishing longitudinal resistivity ρ_L . At sufficiently high magnetic field, the series of QH-to-QH transitions is terminated by a transition to an insulator, marked by a *divergence* of ρ_L .

While insulating and conducting phases are distinguished quite unambiguously by a diverging or vanishing ρ_L (as a function of temperature or system size), the behavior of ρ_H in the insulator can be rather subtle

[2]. Certain theoretical studies of insulators at low B find a *Hall-insulator* behavior [3–6], characterized by a finite, Drude-like $\rho_H \sim B$. Such a Hall insulator has been observed in part of the strong magnetic field (QH) regime as well [7–9]. More recent measurements [10–12] have identified an insulating regime where ρ_H is quantized at the plateau value of the nearby QH state — hence dubbed a *quantized Hall insulator* (QHI). Indeed, a QHI behavior was proven [13] to be a robust feature of an arbitrary network formed by weakly coupled incompressible electron puddles that are in the QH regime at filling factor ν , provided transport on the network is *incoherent* (i.e., governed by *classical* Kirchhoff laws). Recent numerical studies [14,15] indicate that quantum interference actually destroys the QHI: once the localization length becomes much shorter than the dephasing length, ρ_H was found to diverge. While experimental data consistent with such a strongly localized regime are not yet available, the numerical results raised the question whether the QHI

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constitutes a true (zero temperature) phase of a disordered 2D electron system in a strong perpendicular magnetic field. Answering this question within an *analytic* approach designed to gain more insight into localization in high-field insulators was the main motivation for our work presented here.

We choose to model coherent electron transport in the QH regime by a hierarchical random network of chiral QH edge channels that are formed at the boundary of incompressible electron puddles in the sample. The hierarchical structure enables us to construct a 2D generalization of the familiar scattering approach [16] to localization in 1D. This yields scaling expressions for *both* ρ_L and ρ_H . In contrast with the 1D case, such 2D network models (on either a square lattice [17] or a hierarchical structure [18,19]) exhibit a conductor–insulator transition, reflecting the competing tendencies to localization along serial connections of scatterers and delocalization due to parallel connections. We find that, while ρ_H is quantized in the conducting phase where $\rho_L \rightarrow 0$, ρ_H diverges exponentially with system size in the insulator ($\rho_L \rightarrow \infty$).

2. The model and principal results

We consider a hierarchical lattice where bonds represent chiral QH edge channels that are formed around puddles of electron density at filling factor $\nu = 1/m$ with odd integer m . Vertices simulate the scattering that occurs between puddles due to tunneling via saddle points of the smooth disorder potential. The geometry of the elementary network, depicted in Fig. 1, is chosen to mimic a four-terminal measurement setup. Iterative replacement of each single vertex (numbered 1–4) by the elementary network element yields the two-level hierarchical structure that is shown in Fig. 2. The number of elementary vertices present in the n th level hierarchical lattice that is reached after $n - 1$ more iterations can be used as a measure for the area occupied by the system. This defines an effective system size $L = 2^n$.

Transmission amplitudes $t_i^{(0)}$ at the elementary vertices as well as quantum phases acquired when moving around plaquettes (e.g., φ_{14} , φ_{23} , and ϕ in Fig. 1) are assumed to be broadly distributed random variables. In particular, the quantum phases are uniformly distributed. The transmission and reflection probabil-

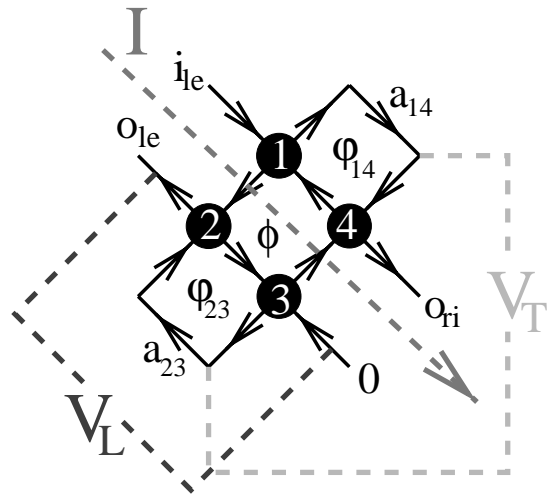


Fig. 1. Elementary network unit consisting of four elementary vertices. Its design is modeled after a four-terminal Hall-measurement geometry. The longitudinal resistivity is given by $\rho_L = V_L/I$. Note that, in general, the Hall voltage is not equal to the transverse voltage drop V_T but rather it is the part that is antisymmetric under reversal of magnetic field at fixed current direction. Hence, we define the Hall resistivity as $\rho_H = [V_T(I, B) - V_T(I, -B)]/(2I)$.

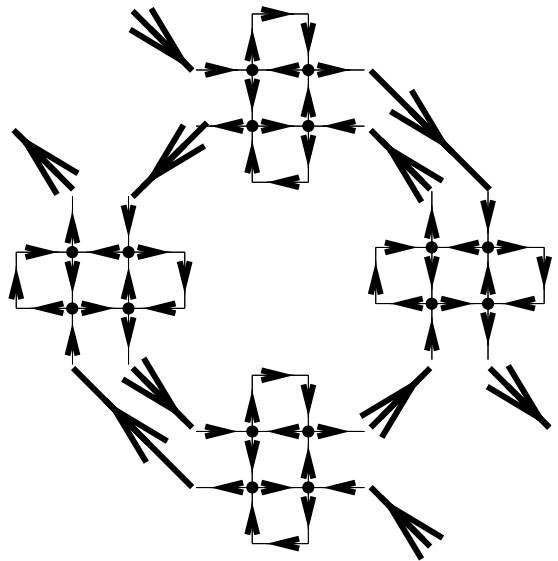


Fig. 2. Hierarchical lattice after the first iteration.

ities of the n th level hierarchical lattice, denoted by $T_i^{(n)}$ and $R_i^{(n)}$, respectively, can be expressed in terms of those of the four $(n - 1)$ th level hierarchical lattices

$\{T_i^{(n-1)}\}_{i=1}^4$ and random phases $\varphi_{14}^{(n-1)}$, $\varphi_{23}^{(n-1)}$ and $\phi^{(n-1)}$. Averaging over all random variables yields recursion relations for $\langle \ln T^{(n)} \rangle$, $\langle \ln R^{(n)} \rangle$, and their standard deviations $\sigma_T^{(n)}$, $\sigma_R^{(n)}$ [20]. These indicate that $T^{(n)}$ and $R^{(n)}$ are approximately log-normally distributed quantities for $n \gg 1$. It is hence possible to define their *typical* values as

$$T_{\text{typ}}^{(n)} = \exp\{\langle \ln T^{(n)} \rangle\} = (\tilde{T}_0)^L, \quad (1)$$

$$R_{\text{typ}}^{(n)} = \exp\{\langle \ln R^{(n)} \rangle\} = (\tilde{R}_0)^L. \quad (2)$$

Here $\tilde{T}_0 \equiv T_0 \exp\{\tilde{\sigma}_T\}$ and $\tilde{R}_0 \equiv R_0 \exp\{\tilde{\sigma}_R\}$, where T_0, R_0 are the typical transmission and reflection probabilities of elementary vertices, and $\tilde{\sigma}_{T(R)}$ depend on the width of their distributions [20].¹

In the same manner, it is possible to obtain typical values of any physical quantity whose logarithm can be expressed as a linear combination of $\ln T^{(n)}$ and $\ln R^{(n)}$. In particular, this is the case for the dimensionless longitudinal resistivity, which is given by $\rho_{L,\text{typ}}^{(n)} = R_{\text{typ}}^{(n)}/T_{\text{typ}}^{(n)}$. Using Eqs. (1) and (2), we find

$$\rho_{L,\text{typ}} = (\tilde{R}_0/\tilde{T}_0)^L. \quad (3)$$

This result implies the existence of two phases; a conducting one for $\tilde{R}_0 < \tilde{T}_0$, and an insulating one for $\tilde{R}_0 > \tilde{T}_0$. The unstable critical point separating the two is defined by $\tilde{R}_0 = \tilde{T}_0$.

To investigate the scaling of the Hall resistivity with L , which is the main focus of our work, we notice that the dimensionless Hall voltage is obtained by antisymmetric combination of the transverse voltages V_T (see Fig. 1) measured at opposite values of magnetic field:

$$V_H^{(n)} = \frac{V_T^{(n)}(I, B) - V_T^{(n)}(I, -B)}{2}, \quad (4)$$

where $V_T^{(n)} = A_{14}^{(n)} - A_{23}^{(n)}$ is given in terms of currents $A_{ij}^{(n)} \equiv |a_{ij}^{(n)}|^2$ flowing in closed plaquettes at the top-level hierarchical structure. The Hall resistivity is then evaluated as $\rho_{H,\text{typ}}^{(n)} = V_{H,\text{typ}}^{(n)}/T_{\text{typ}}^{(n)}$. In the conducting phase, $\rho_{H,\text{typ}}^{(n)} = 1 + \mathcal{O}(R_{\text{typ}}^{(n-1)})$ turns out to be quantized in the thermodynamic limit. In the insulating phase, we express $\rho_{H,\text{typ}}^{(n)}$ in terms of $\langle \ln A_{14}^{(n)} \rangle$, $\langle \ln A_{23}^{(n)} \rangle$, for

which we derive expressions [20]

$$\langle \ln A_{14}^{(n)} \rangle = \langle \ln T^{(n-1)} \rangle, \quad (5)$$

$$\langle \ln A_{23}^{(n)} \rangle = \langle \ln T^{(n-1)} \rangle + 2\langle \ln R^{(n-1)} \rangle + \frac{\sigma_R^{(n-1)}}{\sqrt{\pi}}. \quad (6)$$

This yields an *exponentially diverging* scaling form for the Hall resistivity,

$$\rho_{H,\text{typ}} = [1 - (\tilde{R}_0)^L]/(\tilde{T}_0)^{L/2} \approx (\tilde{T}_0)^{-L/2}, \quad (7)$$

which is the main result of our calculation.

3. Concluding remarks

The result displayed in Eq. (7) is consistent with numerical evidence [14,15] that the typical Hall resistivity of a phase-coherent QH system *diverges* in the insulating regime, though more moderately than $\rho_{L,\text{typ}}$ [given in Eq. (3)]. In the strongly insulating limit ($\tilde{T}_0 \ll \tilde{R}_0$), $\rho_{H,\text{typ}}$ and $\rho_{L,\text{typ}}$ are simply related: one can define a single localization length $\xi = -1/\ln \tilde{T}_0$, in terms of which $\rho_{L,\text{typ}} \sim \exp\{L/\xi\}$ and $\rho_{H,\text{typ}} \sim \exp\{L/2\xi\}$.

In conclusion, we have derived a scaling theory for both longitudinal and Hall resistivities in a hierarchical network model of a QH system. In agreement with previous numerical results, we find that localization effects associated with phase coherence of the electron transport lead to destruction of the QHI phenomenon, and a *true* insulating behavior with both $\rho_{L,\text{typ}}$ and $\rho_{H,\text{typ}}$ diverging.

An intriguing implication of this result is that the QHI behavior, experimentally observed in a rather wide range of parameters, can be considered a signature of a strong dephasing mechanism, due to which transport is dominated by classical processes. A crossover to a transport regime where the quantum effects are observable (and marked by a deviation of $\rho_{H,\text{typ}}$ from the quantized value) is expected either deeper in the insulating phase (at higher B) or at lower temperatures [21,22]. As was pointed out earlier [14], in this regime extracting the typical size of electron puddles from the magnetic field dependence of ρ_L and monitoring the breakdown of the QHI behavior allows for the measurement of the dephasing length.

¹Note that, in contrast to the case of a 1D chain of random scatterers, the typical transmission probability of the 2D hierarchical network depends *both* on the logarithmic average $\ln T_0 \equiv \langle \ln(|t_j^{(0)}|^2) \rangle$ and its variance.

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