(Conference Workbook version)

2

THE UNQUANTIZED QUANTUM HALL EFFECT IN COULOMB DRAG

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ABSTRACT

We show that the Coulomb drag between two weakly coupled quantum Hall systems is a novel probe of their internal dynamics, particularly of the compressible states. The temperature dependence of the drag is sensitive to a frequency and wave-vector regime ($\omega << q$) inaccessible to transport measurements in a single layer. Most strikingly, at the transitions between plateaux it directly measures an exponent η, first discussed by Chalker, that is characteristic of the fractal structure of the critical eigenstates. In contrast, in the $\nu = 1/2$ state in a clean system the drag is identical to that in the zero field Fermi liquid.

1. Introduction

Recently, much interest has focussed[1] on states in compressible regions of the phase diagram of the two-dimensional electron system (2DES) in a strong magnetic field B. In these states, which are observed at the transitions between the plateaux and even denominator fillings, the 2DES exhibits novel correlated behavior *without* quantized values of the conductivites — a state of affairs that Halperin has dubbed the "unquantized" quantum Hall effect (QHE). The dynamics of these states is of considerable interest and indeed is richer than that of the ideal (odd denominator) QH states whose response is dominated by the gap in the excitation spectrum. In this work[2] we show that the phenomenon of Coulomb drag can be used to probe these dynamical properties in an unusual frequency and wave-vector regime, where existing theories can be used to make some striking predictions.

Coulomb drag, first predicted by Pogrebinskii and by Price[3], is the phenomenon whereby a current flowing in one system induces a current/voltage in another, nearby system, even though the two do not exchange particles (a typical geometry is depicted in Fig. 1). The drag is measured by the trans-resistivity ρ_t,

$$\rho_t = \mathcal{E}_u / j_d\,, \tag{1}$$

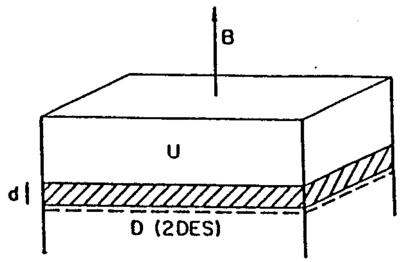

Fig. 1. Schematics of the heterostructure; The shaded area is an insulator.

where $\mathcal{E}_u$ is the parallel electric field induced in U (in open circuit) in response to a current density j_d established in D. Recent experiments, all in $B = 0$, have successfully measured the drag in heterostructures involving a 2DES, separated from another electron or hole system[4]. These were accompanied by a number of theoretical studies[5,6]. The physics of the drag is quite intuitive for it is much like ordinary friction. While a perfectly uniform current carrying state does not exert a Coulomb force on its neighbor, inhomogeneities in the density creates a "corrugated" potential which pushes charge carriers in the other layer in the direction of net current flow. The drag is therefore sensitive to the ability of the electronic states in the layers to support long lived density fluctuations, and thus to their internal dynamics. More formally, Zheng and MacDonald[6] have shown that, to leading order in the interaction V between two 2DESs, ρ_t can be expressed as

$$\rho_t = \frac{\beta\hbar^2}{\pi n^{(u)} n^{(d)} e^2} \int \frac{d^2q}{(2\pi)^2} q^2 |V(q)|^2 \int_0^\infty d\omega \frac{\mathrm{Im}\chi_d(q,\omega)\mathrm{Im}\chi_u(q,\omega)}{4\sinh^2(\beta\hbar\omega/2)}\,, \tag{2}$$

where $\mathrm{Im}\chi_{d(u)}(q,\omega)$ are the dissipative density-density response functions of layer D (U); $\beta = 1/k_B T$, where T is the temperature, and $n^{(d(u))}$ the density in layer D (U).

In the physics of the QHE, density fluctuations indeed play a priviliged role. The QH plateaux typically reflect the presence of incompressible states at odd denominator fillings. Near even denominator fillings one observes compressible behavior, either arising from critical behavior between neighboring plateaux in "dirty" samples[7,8], or from Fermi liquid (FL) states in the cleanest samples[9]. In both cases, problems such as the relative significance of interactions, disorder and their interplay is still a subject of active research. The former case has been extensively studied, within non-interacting models, in terms of delocalization transitions to "metallic" states[7]. Following this approach, Chalker has predicted the possibility of anomalously slow diffusion, characterized by an exponent η (see below), which reflects the fractal structure of the critical eigenstates[8]. The dynamical properties of these critical states are substentially distinct from the FL behavior predicted by ref. 9; however, they are inaccessible to conventional transport measurements. Below we show that interlayer drag can be potentially used as such a probe.

2. Principal results

At a fixed temperature, we find[2] that the dependence of ρ_t on the filling factor ν is qualitatively similar to that of σ_{xx}: it is greatly suppressed deep in the QH phases, and peaks at the transition between them. This behavior follows directly from the suppression of density fluctuations in the incompressible states. Below we focus on the compressible regions, and study the consequence of conjectured properties of the corresponding states.

For non-interacting, disordered systems, Chalker and Daniell[8] have shown that the low frequency response at criticality has the form:

$$\mathrm{Im}\chi(q,\omega) = \frac{(dn/d\mu)\omega\mathcal{D}(q,\omega)q^2}{[\omega^2 + (\mathcal{D}(q,\omega)q^2)^2]}, \quad \mathcal{D}(q,\omega) = D\left(\frac{C\omega}{q^2}\right)^{\eta/2} \tag{3}$$

for $C\omega < q^2$; here $D = 0.087/\hbar(dn/d\mu)$, $C \approx 60\hbar(dn^{(d)}/d\mu)$, where $dn/d\mu$ is the density of states at the band center, and the universal exponent $\eta \approx 0.38$. For $C\omega > q^2$, Eq. (3) is replaced by a conventional diffusive form, i.e. $\mathcal{D}(q,\omega) \approx D$. Inserting this for $\mathrm{Im}\chi_{d(u)}$ in Eq. (2) (assuming two identical layers), we obtain

$$\rho_t \approx 3.1 \times 10^{-4}\frac{h}{e^2}\left(\frac{k_BT}{\hbar Dn}\right)^2\left(\frac{\epsilon}{e^2d(dn/d\mu)}\right)^2\left(\frac{Cd^2k_BT}{\hbar}\right)^{-\eta}, \tag{4}$$

where n is the density, ϵ the background dielectric constant, and d the interlayer distance. Note that we made use of an important property of Eq. (2): for $T \to 0$, the T dependence of the denominator forces the relevant range of ω to vanish, and hence the behavior of ρ_t becomes sensitive to the form of $\mathrm{Im}\chi_{d(u)}(q,\omega)$ for $\omega << q$. This is *in contrast* with dc transport measurements in a single layer, which probe the opposite limit, and consequently is not sensitive to the anomalous diffusion regime. Eq. (4) implies that ρ_t in the critical region is *parametrically* enhanced at low T over non-critical filling factors (at which[2] $\rho_t \sim O(T^2)$), and its T dependence directly measures η. In fact[2], at any arbitrary critical point ρ_t probes an equivalent exponent, that parametrizes anomalous low-frequency dissipation. For a sample of mobility $\sim 10^6\,\mathrm{cm}^2/\mathrm{Vs}$, $n \sim 10^{11}\,\mathrm{cm}^{-2}$, $\epsilon \sim 10$, $d = 200\,\AA$ and $B = 10T$, we estimate $\rho_t \approx 10\,\mathrm{m}\Omega$ at $T \approx 0.1K$.

To compare the above result to the extremely clean limit, in which the transition regions are believed to contain FL states[9], we use the form $\mathrm{Im}\chi \sim \omega/q$ in Eq. (2). We then find $\rho_t \sim T^2$, as in the $B = 0$ case!

3. Discussion

The enhancement of interlayer drag by anomalous diffusion, reflected by the low T behavior $\rho_t \sim T^{2-\eta}$, is in accord with our earlier description of the underlaying physics. The anomalously slow dynamics implied by Eq. (3) ensures that large amplitude fluctuations in the density are maintained for long times ($\mathcal{D}(q,\omega)$ describes the rate of relaxation of such fluctuations). In comparison, ballistic dynamics (such as in a free electron gas) implies $\rho_t \sim T^2$, while regular diffusion enhances the drag at low T by a $\ln T$ factor[6]. As a consequence, at $T \to 0$ the expression in Eq. (4) is expected to overwhelm any contribution to ρ_t from 'conventional' dynamical processes.

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