

Wide Field of View Wavefront Sensing

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Standard adaptive optics systems measure and correct the wave front error along the path to the reference star. Because the measured volume is very narrow, and because the correction is limited to the region of the measured path, the attained field of view is rather small – a few arc seconds. The problem is even more severe when the reference star is not the measured star: either it is another natural star which could be far away or an artificial guide star at tens of kilometers above the telescope. In both cases the measured volume of atmosphere (Figure 1) does not overlap with the observation path to the star and its vicinity, namely a truncated cone whose bases are the telescope and the star field. Even for bright objects, where the measurement is accurate, the corrected isoplanatic angle cannot extend beyond the few arc seconds limit. One would like to have wider fields, of arc minutes and more, over which turbulence can be corrected.

A wider field of view can be achieved by multiconjugate adaptive optics. In this method, the atmospheric turbulence is measured at various elevations and corrected by using several optical elements, usually conjugate to the most offending layers. The idea was proposed very early on by Dicke [1975] and by McCall and Passner [1978] and became possible with the suggestion of laser guide stars by Hudgin in 1980 and Feinlieb in 1981 [Fried 1992], and by Foy and Labeyrie [1985] and Beckers [1988, 1989]. However, in the same manner that technical difficulties delayed the employment of adaptive optics in general, multi-conjugate adaptive optics has been slow in application, and only two initial experiments have been performed in this direction [Murphy *et al.* 1991, Neyman and Thompson 1995]. Like the rest of adaptive optics, there are some issues that will have to be addressed before the method is put to use. Since no adaptive optics system has yet been designed and constructed with wide field correction, most of the discussion to follow will concentrate on principles, identified problems and suggested solutions.

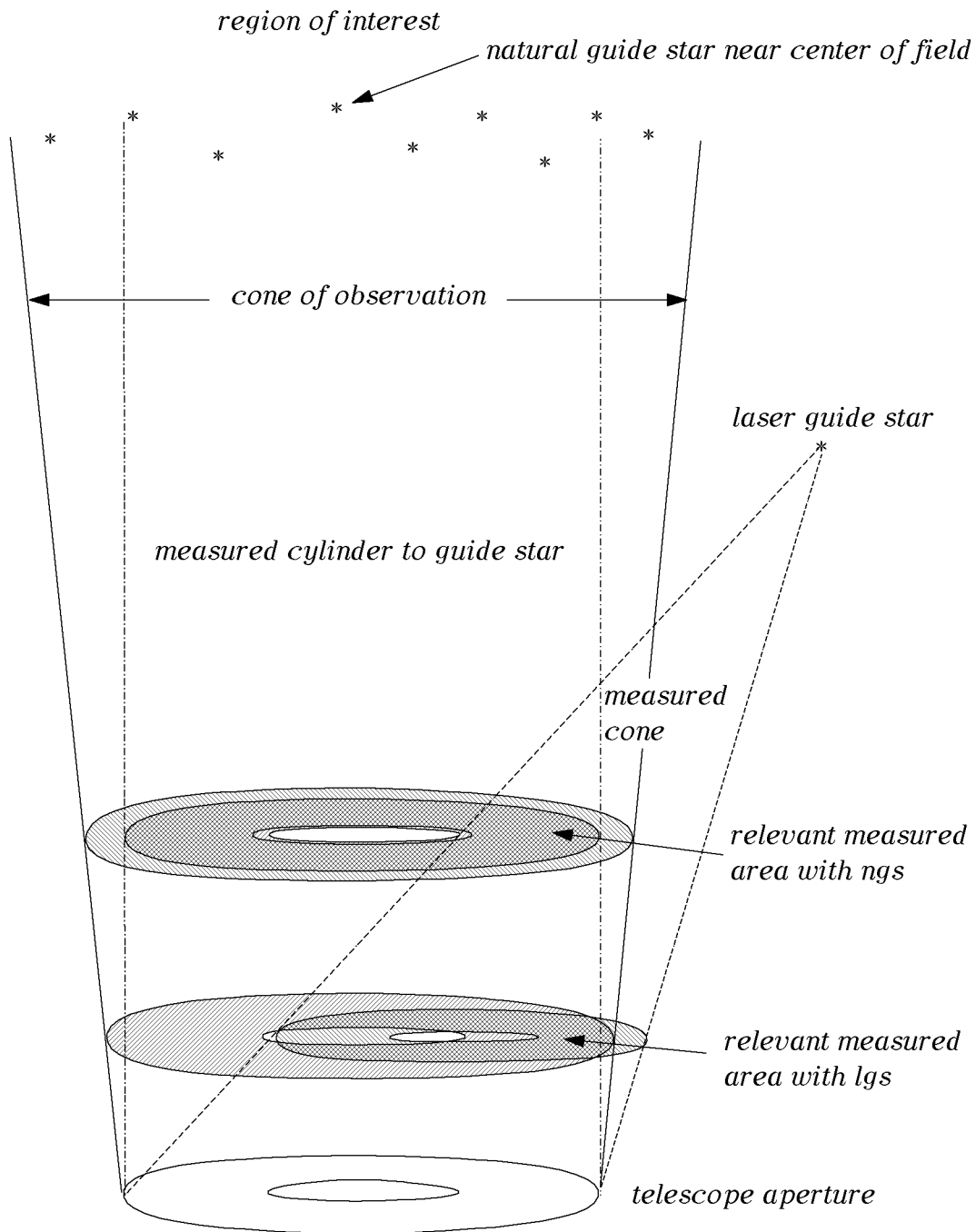


Figure 1. The measured volume of atmospheric turbulence is smaller than the volume to be corrected for a wide field. The overlap between measurement and required correction (crosshatched) is shown schematically for two atmospheric layers: using a natural guide star (ngs: top) and a laser guide star (lgs: bottom). If the deformable mirror is conjugated to this layer, it can only correct it partially. If the area outside the mirror is blocked, it leads to vignetting.

1. Separation of the atmosphere into thin layers.

Let us assume we have a correcting element, such as a flexible or segmented mirror working in reflection (Chap. 5), or a liquid crystal phase modulator [Love 1993], in transmission mode (Chap. 9). To simplify the discussion we shall refer to all wave front correctors as deformable mirrors. Now in order to have maximum effect these mirrors have to be conjugated to the most offending atmospheric layers, or be placed strategically, so as to maximize their impact on the performance of the system. Thus, the effective turbulence height profile $C_n^2(h)$ [Roddier 1981] has to be known. Because wave front measurements tend to be noisy, the sub-aperture size must be as large as possible so as to collect more light (Chap. 4). However, it is limited by the size of r_o [Roddier 1981] for each layer. Thus, knowledge of the actual r_o of the separate layers is also useful, as well as the wind speed and direction in the various layers.

Is it at all possible to locate optimal positions for the mirrors? Is atmospheric turbulence continuous, or is it layered? These questions have various answers, according to telescope site and specific constraints. Older measurements of the turbulence [Bufton *et al.* 1972] led to rather accurate models (e.g. Hufnagel-Valley, [Hufnagel 1974]) by assuming continuous turbulence [Coulman 1985, Beland 1993]. However, Troxel *et al.* [1994] were able to show that the atmosphere can be modelled adequately by using only four distinct layers. They were able to achieve an atmospheric optical transfer function accurate to within one percent of that of a continuous atmosphere (Figure 2). Careful inspection of the older measurements of $C_n^2(h)$ show [Bufton *et al.* 1972, Coulman 1985, Beland 1993] that the atmospheric turbulence profile (as a function of height) is indeed rather intermittent. Other measurements [Vernin and Roddier 1973, Rocca *et al.* 1974, Roddier and Vernin 1977, Azouit and Vernin 1980, Sarazin 1987, Caccia *et al.* 1987, 1988, Tallon *et al.* 1992a, Roddier *et al.* 1993, Irbah *et al.* 1993, Sivaramakrishnan *et al.* 1995, Vernin and Muñoz-Tuñón 1994, Acton *et al.* 1996] seem to indicate that at very good sites the number of layers may be as small as two or three, and it becomes larger for worse sites. However, even at a very good site, modelled with only one or two dominant layers, there is some residual turbulence, not included in these layers, which has to be dealt with [Racine and Ellerbroek 1995].

Thus, there are three approaches that the designer of a multi-conjugate adaptive optics can take, once he had acquired a good knowledge of the turbulence profile $C_n^2(h)$, of $r_o(h)$, and of $v_{wind}(h)$:

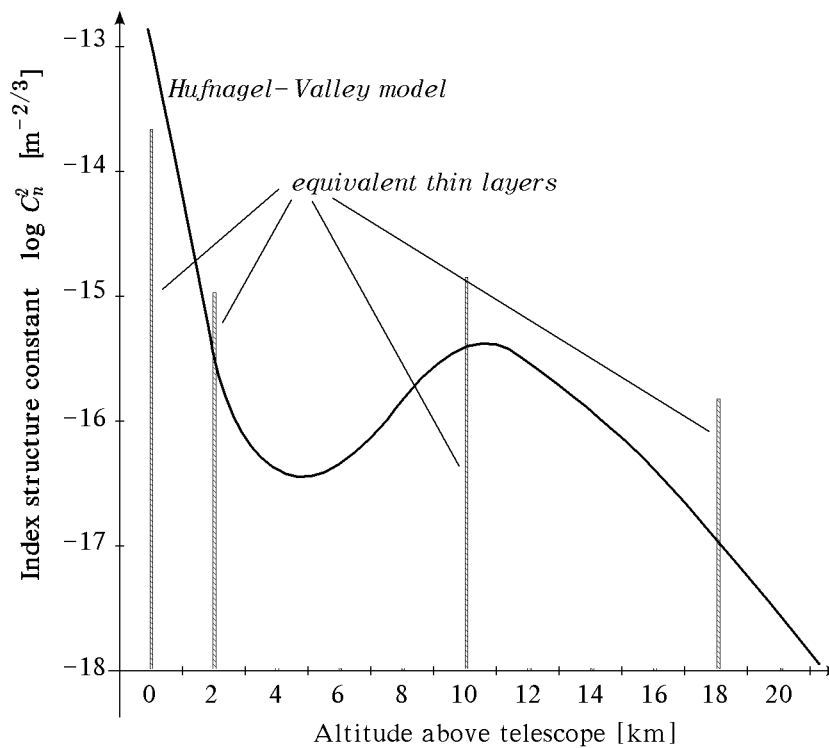


Figure 2. Atmospheric turbulence is layered, as various measurements imply. Even very smooth models which represent it properly are still equivalent to very few layers as shown by experiment [Tallon *et al.* 1992a] and simulation [Troxel *et al.* 1994].

- a. Fit a continuous model to $C_n^2(h)$; optimize the conjugate planes for the deformable mirrors. This approach is good for the cases where the seeing is usually bad and the atmosphere cannot be well described by a set of few layers. It is also applicable when it is not practical to vary the adaptive optics system, and an average model of the atmosphere is preferable.
- b. Fit a few-layer model to $C_n^2(h)$; conjugate the deformable mirrors to these fitted layers. Here it is assumed that all the turbulence is concentrated in the corrected layers.
- c. Fit conjugation planes for the deformable mirrors to $C_n^2(h)$ (not necessarily at the roughest layers) [Racine and Ellerbroek 1995, Avila *et al.* 1998]. Parts of this task can be achieved by neural networks [Monterra 1996].

Cost functions for optimizing the conjugation planes of the deformable mirrors are some combination of the following:

- a. Spatial band of the observed object. For example, when one is interested only in coarse or fine details of the object, or a specific orientation, such as when looking for extra-solar planets [Angel 1994]. In this case, objects are not at the highest spatial frequencies, but they are very different in intensities. The adaptive optics point spread function should be optimized so as not to have side lobes which might scatter light from the main star to its faint companion. Another example is spectroscopy, where one needs to get as much light into the slit, but it might not be as important to minimize the image size in the lateral direction.
- b. Spectral band. Turbulence is much worse in the visible regime as compared to the infra red regime. Thus, optimization should be easier and requirements lighter when dealing with the infra red observations.
- c. Temporal band. Usually the turbulence sets the time scales in the servo loop. Lower turbulence tends to be faster than higher one, except when jet streams occur (usually at very good sites like Hawaii). However, when observing a fast changing object like the sun, difficulties might arise. This is because the finest (and most interesting) details evolve the fastest, and locking on to them might be difficult [Acton *et al.* 1996]). At very low light levels the long integration times might be a limiting factor.
- d. Isoplanatic angle. Usually an adaptive system achieves high resolution at the center of the field and low resolution at the edges; instead, one might require the same medium resolution over the whole field. In such cases it will usually be the lower turbulence which has to be corrected, since it is shared for all field objects (Figure 3) [Acton *et al.* 1996].
- e. Strehl ratio. This parameter was chosen by Racine and Ellerbroek [1995] and by Wilson and Jenkins [1996]. However, the Strehl ration is usually defined for the center of the field, where it is the highest. Alternative, one can use an average, lower Strehl ratio for the whole field. Ellerbroek [1994] discussed using the residual mean-square phase distortion and the associated optical transfer function.
- f. Location and availability of natural and laser guide stars. Natural stars (if not the observed star itself or a glint from a man-made object) occur at random places which cannot be chosen (apart from moving asteroids [Ribak and Rigaut 1994]). Laser guide stars are limited by available power and hence by number (Section 2).
- g. Mechanical limitations. The optical train might not have enough room to accommodate all possible locations of the conjugate mirrors. This is especially true if the conjugate planes are too close to each other, and bulky deformable mirrors are used for wave front correction (Section 3).

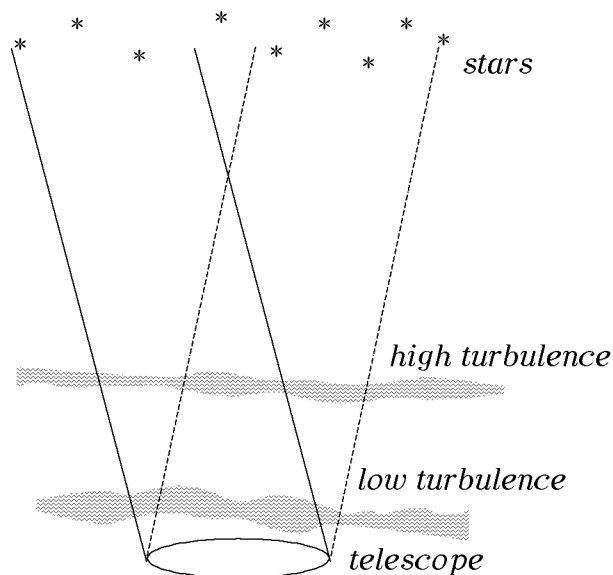


Figure 3. Different parts of the atmosphere are traversed by beams from various stars in the field. A common patch of the lower atmospheric layers is crossed by beams from all stars, and its correction will reduce the total phase error and the Strehl ratio for all these beams.

- h. Optical design. Various factors might lead to limitations in the optical design of the system. Such factors might include variable magnification between conjugate planes, vignetting (Figure 1), the choice of observing equipment, such as a spectrometer or a camera, etc. (Section 3).
- i. Servo loop limitations, electronic and computational. Ellerbroek *et al.* [1994] examined carefully some different choices for servo loop realizations.

Notice that the turbulence profile could change from night to night and the relative importance of the layers might shift during the observation [Racine and Ellerbroek 1995, Avila *et al.* 1998]. This might require a continuous adjustment of the conjugation planes, a complex mechanical task (Section 3).

An adaptive optics systems operating on the principles discussed above still has to be constructed. However, some simple designs involving multiple correction elements are already being implemented. The simplest of all are these systems has a tip-tilt mirror separate from the deformable mirror, usually because the latter has a limited travel range. Currently, these two mirrors are conjugated to the telescope aperture. However, Thompson suggested that it might be better to conjugate them to different heights [Richardson 1992]. The best arrangement should be

decided according to an optimization scheme as described above.

In one option currently under development the secondary mirror of the telescope is also a deformable mirror [Lloyd-Hart *et al.* 1996]. It cannot be conjugated to any atmospheric layer, but a second corrector (perhaps only for tip and tilt) might be useful if conjugated to another layer. The other option is to use a Gregorian design for the telescope, in which case the secondary mirror can be conjugated to an atmospheric layer [Beckers 1993].

2. Measurements of the various layers. Natural and artificial guide stars.

The measurements of the turbulence elevation profile $C_n^2(h)$ and the corresponding $r_o(h)$ and $v_{wind}(h)$ are usually statistical. However, to achieve a wide field of view one must measure the instantaneous values that wave fronts take as they propagate down the atmosphere, along and near the optical axis of the telescope (Figure 1). In essence, this is a tomographic measurement of the refractive index of air inside the relevant region of space. In the astronomical case there are some factors which make tomography simpler than the general case:

- a. Turbulence layers tend to lie in horizontal planes, so the three-dimensional volume can be simplified to a limited set of parallel planes.
- b. Since the statistics of turbulence usually obey Kolmogorov's law, each turbulence layer needs be measured only up to a resolution of $r_o/2$ of that layer. The same applies to the temporal resolution – one can integrate up to $(r_o/v_{wind})/2$ of each layer (more accurately, up to the Greenwood frequency; see Roddier [1981]).
- c. Again, because of Kolmogorov's spectrum, unruly solutions can be easily voted against. Alternatively, constraining the solutions to Kolmogorov's limitations may make the calculation faster.
- d. Because of Taylor's hypothesis of frozen flow, earlier measurements of the turbulence layer provide us with some prior knowledge of the current shape of the wave front [Dicke 1975, Hudgin 1977, Lukin and Zuev 1985, Jorgensen and Aitken 1992, Schwartz *et al.* 1994].

Notice, however, that some evidence shows that the lower layers might not obey Kolmogorov's 5/3 law [Bester *et al.* 1992], perhaps because of heat sources and sinks at the ground level, not accounted for in Kolmogorov's theory.

On the other hand, tomography is made more difficult because

- e. The amount of light available is usually not sufficient. Natural and artificial guide stars provide too little flux for wave front measurements to be accomplished without excessive noise.

- f. The distribution of available light sources is random, too sparse, and arbitrary if one uses only natural stars, or not high enough, if laser guide stars are employed. If laser power is limited, then there might be fewer beacons than required.
- g. The sensors are all at the bottom of the telescope, and can sample only that section of space visible from this location. The lasers also emanate from the vicinity of the telescope. The sensors and the lasers can hardly be distributed in other locations (for an exception, see Ragazzoni [1996]). This limits very much the variability required for tomography.
- h. The measurements must be solved to provide the wave fronts at a rate faster than the atmospheric changes (the Greenwood frequency), for the corrections to be effective.
- i. The global tilt of the wave front accumulated over all layers (or for each layer separately) cannot be determined without employing some special means, such as two-color laser stars [Foy *et al.* 1995] or distant lasers [Ragazzoni 1996]. Ragazzoni and Rigaut [1998] suggest overlapping a natural star and one laser star to find the global tilt.

Wave front sensing is very crucial for adaptive optics, and even more so for multiconjugate adaptive optics. A multitude of methods has been proposed, and new ones are still required. Once it is grasped that atmospheric tomography is the issue at hand, it also becomes apparent that a diversity of sources and a diversity of detectors is required to solve the problem.

The first suggestion of multi-conjugate adaptive optics [Dicke 1975] involved only a single guide star. Making use of a single phase contrast wave front sensor and former measurements, alternate corrections are made at two mirrors conjugate to the aperture and a low layer (Figure 4). This takes care of the cylinder of turbulence between the telescope and the star (see also Angel [1992]). Even this simple approach shows the attraction of multi-conjugate adaptive optics: suppose that only the layer near the telescope (boundary layer) is corrected successfully. Then all other sources in the field will now be affected only by higher turbulence (Figure 3). Correction of these high layers can be achieved by locking on other stars, further off in the field. This is because all the stars in the field of view share the bottom layer turbulence. Alternatively, correcting the higher turbulence alone may result in larger isoplanatic angles but worse Strehl ratio.

The suggestion made by Hudgin in 1980 and by Feinlieb in 1981 to use laser guide stars for military purposes [Fried 1992], and a similar but independent proposal for astronomy [Foy and Labeyrie 1985] made the guide star problem simpler: there was no need any more to rely on the observed star itself as the reference beacon. It was grasped from the start [Foy and Labeyrie 1985, Tallon and Foy 1990, Gardner *et al.* 1990, Sandler 1992, Shamir and Crowe 1992] that many sources could be hung in the sky, and their geometrical distribution was in the hands of the user, limited only by technical constraints.

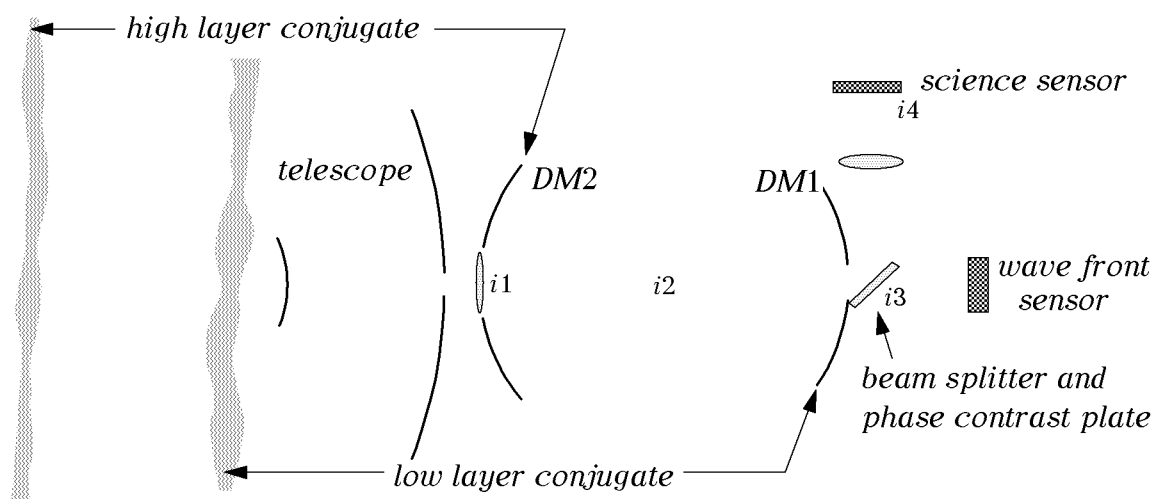


Figure 4. Correction of atmospheric layers by a single guide star and a single wave front sensor operating as a phase contrast detector [Dicke 1975]. The telescope images the object at i_1 and a lens there images the lower turbulence on the first deformable mirror ($DM1$). This mirror re-images the field at i_2 . A second deformable mirror ($DM2$, conjugate to the turbulence at 5 km) relays i_2 to i_3 . Here it is split to the wave front sensor and to the science sensor at i_4 .

It seems that a web of guide stars, Rayleigh or sodium ones, are necessary to solve the tomographic problem successfully [Foy and Labeyrie 1985, Gardner *et al.* 1990, Tallon and Foy 1990, Jankevics and Wirth 1992, Sandler 1992, Shamir and Crowe 1992, Shamir *et al.* 1993, Tyler 1994, Ellerbroek 1994, Baharav *et al.* 1994, 1996, Ragazzoni *et al.* 1999]. However, it was found that a smart approach will have to be taken, and even then the system might not be efficient or effective if the atmosphere cannot be modeled by thin layers alone [Fried 1995].

If one wishes to have a wider isoplanatic patch, and is willing to give up on very high Strehl ratio, then a Rayleigh beacon might be sufficient. This is because such a beacon samples the lower atmosphere only (Figures 1, 3), and that section of the atmosphere is responsible for most of the errors common to the wider field of view [Ellerbroek 1994, Tyler 1994]. This argument was proved by measurements [Christou *et al.* 1995].

Foy *et al.* [1989], Tallon and Foy [1990], and Tallon *et al.* [1992a,b] show that the number of sodium guide stars must be equal or greater than the number of thin atmospheric layers. Three laser guide stars might be sufficient; four are definitely enough (Figure 5). Gardner *et al.* [1990] and Welsh and Gardner [1991], using arguments of isoplanicity with a continuous (non-layered) atmosphere, arrive at similar numbers for sodium stars and much higher numbers (in the hundreds) for Rayleigh stars.

Approaches using Rayleigh stars for solution of the lower layers and sodium stars for the higher layers [Shamir and Crowe 1992, Shamir *et al.* 1993, Ellerbroek 1994, Tyler 1994, Baharav and Shamir 1995] were shown to increase the field of view. Baharav and Shamir [1995] found out, however, that the residuals from the Rayleigh measurement are comparable to the sodium measurement alone, and might make the Rayleigh beacons redundant.

What is the optimal placement of the guide stars? If the number is low (three to four, according to Tallon and Foy [1990] and Gardner *et al.* [1990]), then the guide stars should be placed around the rim of the telescope field of view (Figure 5). Square arrays with more beacons were considered by Tyler [1994] and Ellerbroek [1994]. In these studies, Rayleigh stars were placed under the corresponding sodium stars. Still, there is no study that compares the various arrangements. These placements depend strongly on the location of the wave front sensors, what volume of space they measure, and whether this volume is shared with the other sensors.

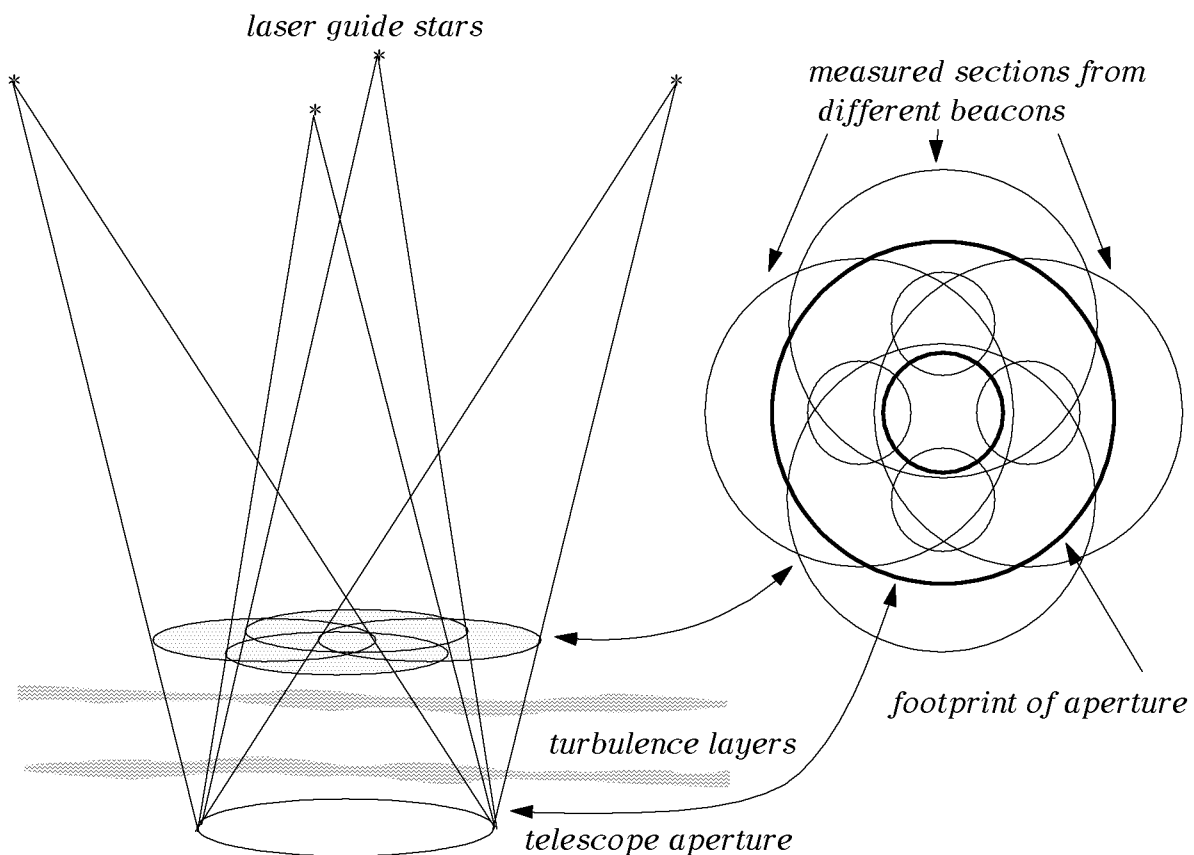


Figure 5. Placement of four laser guide stars. At the top of the atmosphere (about 20 km), the cross-sections of the four cones overlap slightly to cover a contiguous area of the layer. Further down their redundancy (multiple overlap) increases.

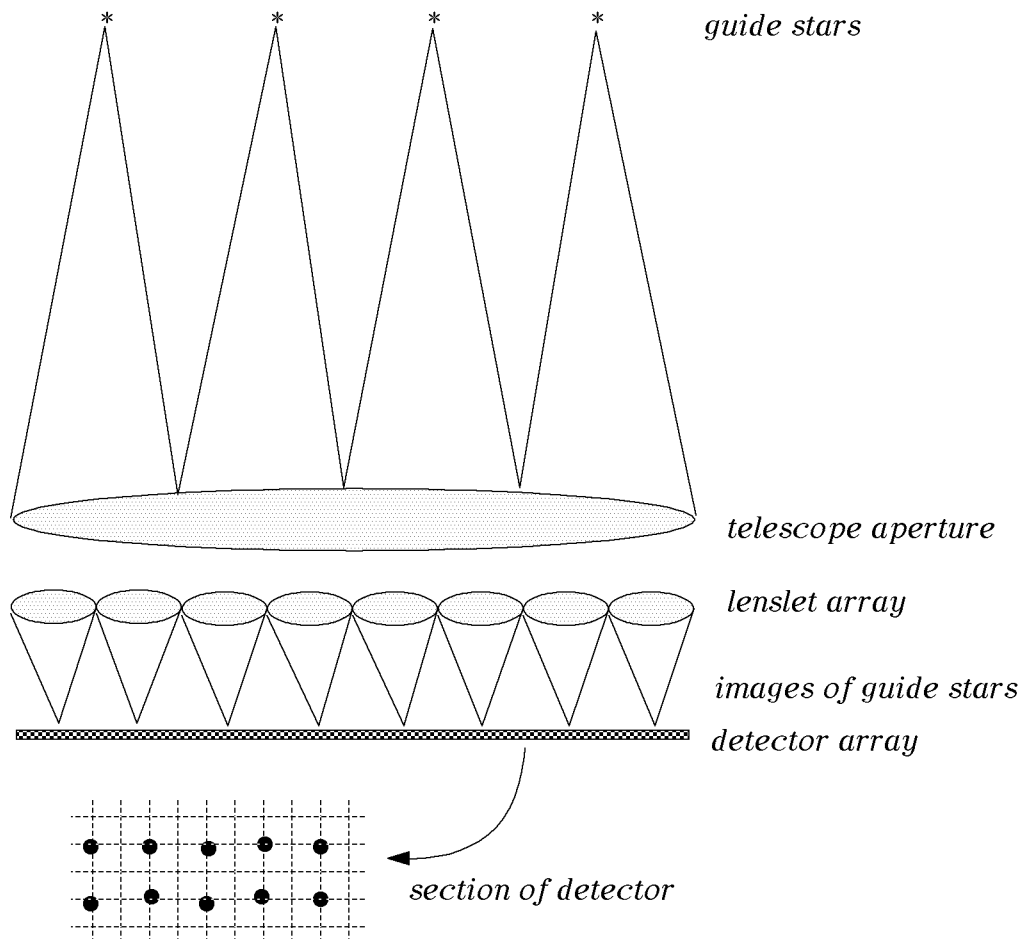


Figure 6. The telescope aperture is imaged on the Hartmann-Shack sensor. Each lenslet has only one guide star in its field of view. The images should fall near the corner of four pixels in detector array for better position sensing and for reduction of the number of pixels. The measured sections of the wave fronts are stitched together in software.

The location of the wave front sensors is different for the different schemes. One approach is to make each sensor measure a wave front from each laser guide star, through a section of the telescope aperture (Figure 6). This samples the turbulence in nearby sections, which are then combined in a process dubbed butting or stitching [Herrmann *et al.* 1992]. This is not a very efficient process, because the separate patches have different tilts which cannot be measured [Sasiela 1994]. Tyler [1994] literally widens the basis of the earlier approaches. He shows the benefits of having each sensor measure the full cone from each star to the whole aperture; sharing the lower part of the atmosphere brings about tomography of the atmosphere (Figure 7). Neyman

and Thompson [1995] proposed and started testing the idea of using part of or the whole aperture for sending up the laser beams as well as for sensing them through parts or over the full aperture.

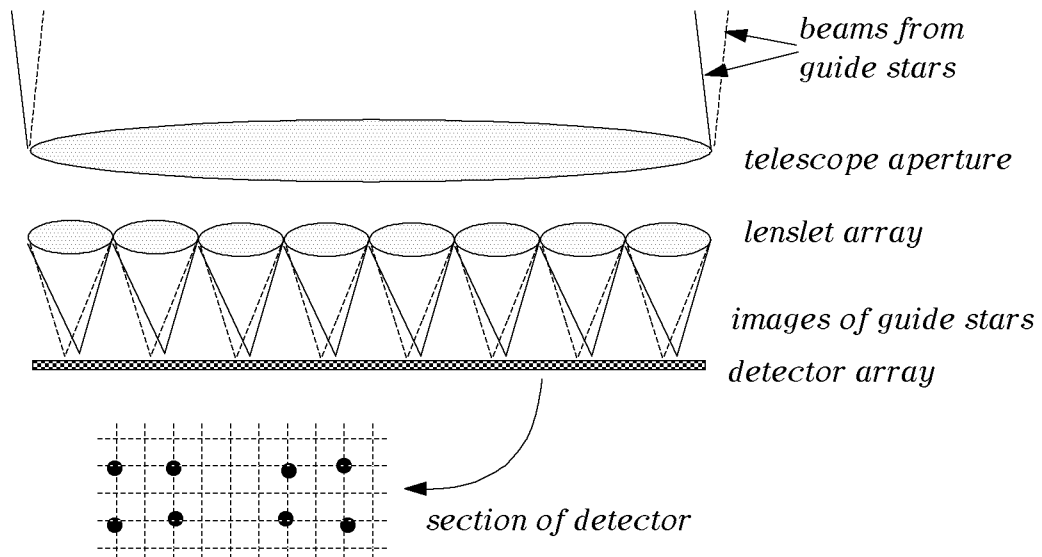


Figure 7. The telescope aperture is imaged on the Hartmann–Shack sensor. Four guide stars create four images near each lenslet focus on the detector array. Notice unused pixels when the foci of the lenslets are far apart, as compared to the fields of view of the lenslets.

How should one extract the phases of the atmospheric layers from the tomographic measurements? The approach proposed initially by Beckers [1988] was to shift and average the wave front sensor measurements as they propagated down the turbulence through different paths. This is justified since the phase differences between the various measurements are rather small [Shamir *et al.* 1993]. Tallon and Foy [1990] and Tallon *et al.* 1992a,b) were more specific. They showed that a set of equations could be set up for each guide star and for each layer, with the appropriate shifts for the different layers. They assume that there are K turbulence layers at elevations $h(k)$ ($k = 1, 2, \dots, K$), and the guide star is at altitude H . The wave front phase at layer k and coordinate \mathbf{r} is $\psi(k, \mathbf{r})$, and its propagation to the ground creates a wave front $\varphi(k, \mathbf{r})$ which is actually a convolution with a harmonic function $\eta[h(k), \mathbf{r}] = \eta(k, \mathbf{r})$ [Roddier 1981]:

$$\varphi(k, \mathbf{r}) = \psi(k, \mathbf{r}) * \eta(k, \mathbf{r}) . \quad (1)$$

Because of geometric effects, one measures a linear combination $G(m, k)$ (essentially scaling and shifting) of the elements of $\varphi(k, \mathbf{r})$ which depends on the location of the m th guide star

$$\varphi(m, k, \mathbf{r}) = G(m, k) \varphi(k, \mathbf{r}) \{1 - h(k) / H\}^n, \quad (2)$$

where the last factor arises because wave front sensing measures the n th derivative of the phase (the phase itself, $n=1$, its gradient, $n=2$, or its curvature, $n=3$). The measurements for all the guide stars are

$$M(m, \mathbf{r}) = \sum_{k=1}^K G(m, k) \varphi(k, \mathbf{r}) \{1 - \frac{h(k)}{H}\}^n . \quad (3)$$

These equations have to be inverted for $\varphi(k, \mathbf{r})$ and deconvolved from the atmosphere (Eq. 1) to yield $\psi(k, \mathbf{r})$, the required phases. Johnston and Welsh [1992, 1994] suggested to solve these equations by least squares fitting. Ragazzoni *et al.* [1999] solved directly for the Zernike modes.

Sandler [1992] proposed to use an ordered array of beacon laser spots spaced so as to have matching fringes in the shearing interferometer wave front sensor (see Chap. 5). The results of the experiment were inconclusive, but the idea brought about an opposite idea: using beacon laser fringes over tens of meters to match a set of Hartmann–Shack sensors. Baharav *et al.* [1994, 1996] describe the scheme as a means to separate lower and higher turbulence layers. The projected aperture of the telescope is broken into a number of Hartmann–Shack lenslets, each facing most of the fringe pattern in the sky (Figure 8). As in the standard sensors, the whole fringe pattern will shift with slope errors, mostly contributed by the low-lying turbulence. At the same time, the images of the fringes will suffer from distortion from high turbulence. Thus the phase errors inside each subaperture can be traced by high-pass filtering, and added up with the neighboring subapertures to yield the high turbulence.

The main disadvantage of the multiple-beacon schemes is their requirement for high power lasers. Creation of many point beacons merely multiplies the requirements from a single beacon, and makes the projection system cumbersome. Either three to four lasers will have to be employed in parallel, with required power of approximately 60–100 Watts. Alternatively, the beam will have to be scanned across the sky (multiplexed stars) at a high rate. Using very conservative calculations, the fringe method requires 300–500 Watts of laser power, but it has a rather simple projection system, that of a simple interferometer (Figure 8). Ribak [1998] suggested replacing the laser fringes with visible plasma fringes, created by interference of radio beams.

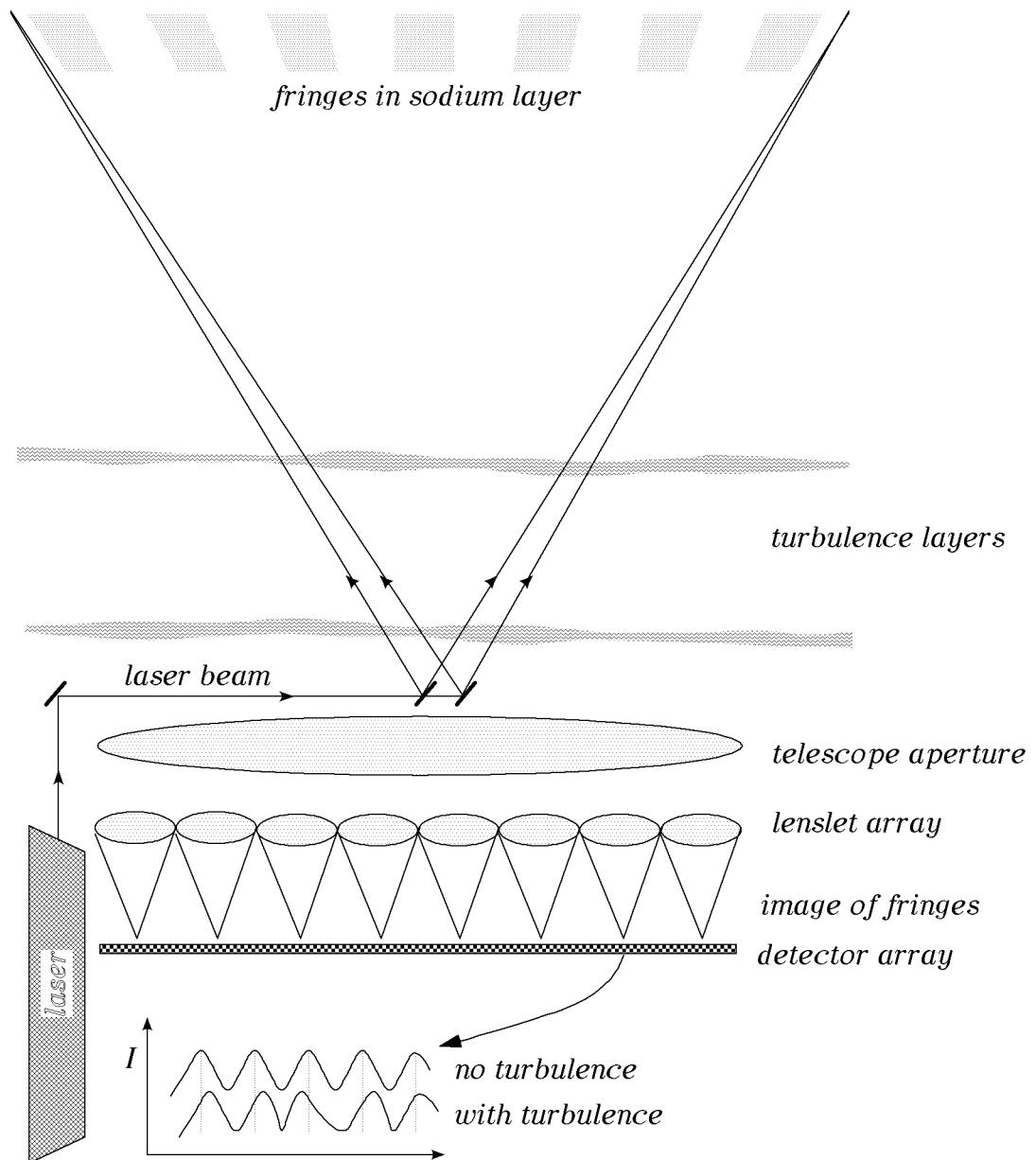


Figure 8. The telescope aperture is imaged on the Hartmann-Shack sensor. Most of the fringe pattern is imaged at each lenslet focus on the detector array. The fringe pattern is shifted as a whole because of turbulence at the low atmosphere, since the conjugate lenslet faces only a small section of it. At the same time the fringes are distorted by the high atmosphere inside the large field of view of the lenslet. These effects (global shift and distortion) also exist for the few-guide-star case (Figure 7).

Acton *et al.* [1996] applied phase-diversity methods [Gonsalves 1994] on solar features to separate low from high wave front aberrations. This method, using simple images at focus and out of focus, is rather slow in processing. Although not fully successful, their results are very encouraging. They show that even low-contrast features, which evolve in time, are sufficient to tell apart the different layers. Love *et al.* [1996] constructed a whole adaptive system using phase diversity, still not at the high rate required by the atmosphere.

Another result of Acton's research on the sun is that even low contrast features are enough for wave front sensing. For night time astronomy, there might not be a special need for a spatial arrangement of laser stars or laser fringes. Papen *et al.* [1996] stress the inhomogeneity of the sodium layer and of light scattered off the sodium. Simple sodium lamps (such as those used for street lighting) might be sufficient to illuminate a very large section of the sky (see a similar approach by Wirth and Jankevics [1992]). Laser light is much more collimated than incoherent light, but it might be possible to use concentrators to get sufficient amount of light in the relevant area. The natural clumpiness of the sodium will result in inhomogeneous back scattering, which could be sufficient for wave front sensing. If the returned intensity is still too smooth, radio beams could be employed to modify it. For example, a radio interferometer could modulate spatially the illuminated sodium layer and create fringes in the returned light [Ribak 1998].

Other means were proposed to measure the different layers. Curvature sensing (Chap. 4) relies on intensity variations down the beam from phase variations. But that effect also means that intensity variations at the aperture of the telescope, better known as scintillations, are related to phase variations at the high atmosphere, kilometers above the telescope. This prompted Ribak [1994, 1996] and, independently, Glindemann and Berkefeld [1996] to propose to invert these intensities to find the original phases. Intensity variations are measured anyway by wave front sensors and discarded as noise. Instead, they can be used for this purpose, especially for brighter sources and stronger turbulence.

The placement of the wave front sensor, at a plane conjugate to a specific layer, is significant. By moving the wave front sensor up and down the optical train, it is possible to cancel scintillation effects in the layer conjugate to the measured one [Bregman *et al.* 1991, Fuchs *et al.* 1994]. But another advantage which emanates from these works is that two curvature sensors, placed at the conjugates of two significant layers, will each measure the other layer simultaneously. Notice how similar this now becomes to Dicke's method [1975].

3. Optical design.

How big a field of view can we expect with current technology? It turns out that, like in so many other cases, we might be limited more by the detectors than by other elements. Consider the following one-dimensional example: We have electronic cameras (charge-coupled devices, for example) comprising $N = 8,192$ pixels (in one or a few tiled cameras). Suppose also that we have a $D = 5$ m telescope. At the wave length of $\lambda = 0.6 \mu\text{m}$ and at maximum resolution, each pixel will see one-half the maximum angular resolution or $0.61 \times 6 \cdot 10^{-7} / 5 = 0.0732$ microradians eaChap. The whole field of view will be $8192 \times 0.0732 = 586$ microradians or two arc-minutes. Larger fields can thus be achieved only by lowering the resolution per pixel, using larger cameras or smaller telescopes. Somewhat larger fields of view (with fewer pixels) might be required by the wave front sensors which need to measure slightly beyond the edge of the projected telescope aperture. The angular size of the field means that laser guide stars at the sodium layer extend over approximately 60 m, and the measured – and corrected – patch at 15 km elevation is about 14 m.

While planning an adaptive optics system, certain issues arise that might become more complex with a multiconjugate system. In some cases the multiconjugate system is a simple extension of the single conjugate system with similar requirements (for example, that the whole system could be pulled out to allow for wide field imaging at low resolution without the benefit of adaptive optics). In other cases special issues will have to be answered. Richardson [1992, 1994] has raised some important subjects, such as the quality of the images of the conjugate planes when relayed down the optical train. The tendency to use reflective (rather than refractive) optics, brought about by the need for good infra red imaging, is very demanding. For this regime, off-axis paraboloids and hyperboloids have to be considered, with their severe off-axis aberrations.

Other issues stem from the requirement for conjugation to various – and variable – atmospheric turbulence planes. Variations of the elevation of the turbulence layers over minutes [Racine and Ellerbroek 1995, Avila *et al.* 1998] might require fast changes. Also, the varying size of r_o requires zooming capabilities on top of the re-imaging capabilities (assuming that the deformable mirror has a fixed geometry with a limited number of elements). At the same time both the scientific detector and the wave front sensor or sensors should stay at the same location or be moved in unison (to within a fraction of a pixel!). The movement of the optical elements has to be designed so as not to have them interfere with each other's path [Richardson 1994].

Wilson and Jenkins [1996] have investigated theoretically the effect of conjugation to different layer heights. Most adaptive optics systems today tend to put the deformable mirror at a

position conjugate to the aperture of the telescope. The study shows that this is useful only in spectroscopic measurements, when one wishes to maximize the amount of energy in the slit. For imaging applications, a much better solution results when the deformable mirror is conjugated to the worst layer, usually higher up. The main disadvantage of this method is that with a single guide star there is vignetting (Wells [1995] and Wells *et al.* [1996]). The degree of overlap is further decreased when using a laser guide star. In this case not only is the area of the higher layer smaller than that of the aperture, but it is also scaled differently (Figure 1). Finally, there is also the problem of mismatch (both in registration and in scale) between the wave front sensor and deformable pixels when conjugated to different layers. It seems that a large number of these problems can be solved by oversampling the turbulence in four dimensions: outside the aperture, at different elevations, and at former time steps. Spatio-temporal prediction using the fractal nature of the wave fronts [Schwartz *et al.* 1994] should replace all but the highest missing frequencies and those upwind from the guide star(s).

Another issue is the order of the correcting elements. When turbulence is weak, the geometric approximation (that beams only bend, but do not diffract or cross) is valid, and the phase errors from the different layers add linearly [Roddiier 1981]. Thus they can be subtracted linearly without regard to which conjugate layer is corrected first. If turbulence is not so weak, it might be better to conjugate and correct first the lower atmospheric layers, and then reimage and correct the higher ones [Dicke 1975, McCall and Passner 1978]. In this manner the errors are undone in opposite direction to the order of their occurrence [Johnston and Welsh 1992, 1994].

How should we deal with objects that are not at zenith? In such a case, the atmospheric layers do not lie normal to the direction of observation, and a corresponding tilt to the deformable mirror needs to be designed (Figure 9). This problem might not have a proper solution, and might limit wide field adaptive optics to the vicinity of the zenith.

4. Future development.

Multiconjugate adaptive optics is considered as the next step after standard adaptive optics has proven its utility, reliability, and usefulness for the astronomer. Only when prices drop and more experience and confidence are gained, will there be room for expansion to more laser guide stars, more sensors and more mirrors.

Until full multiconjugate adaptive optics is available, other methods will have to be employed. Between these, usage of speckle methods for improvement of resolution on the edges of

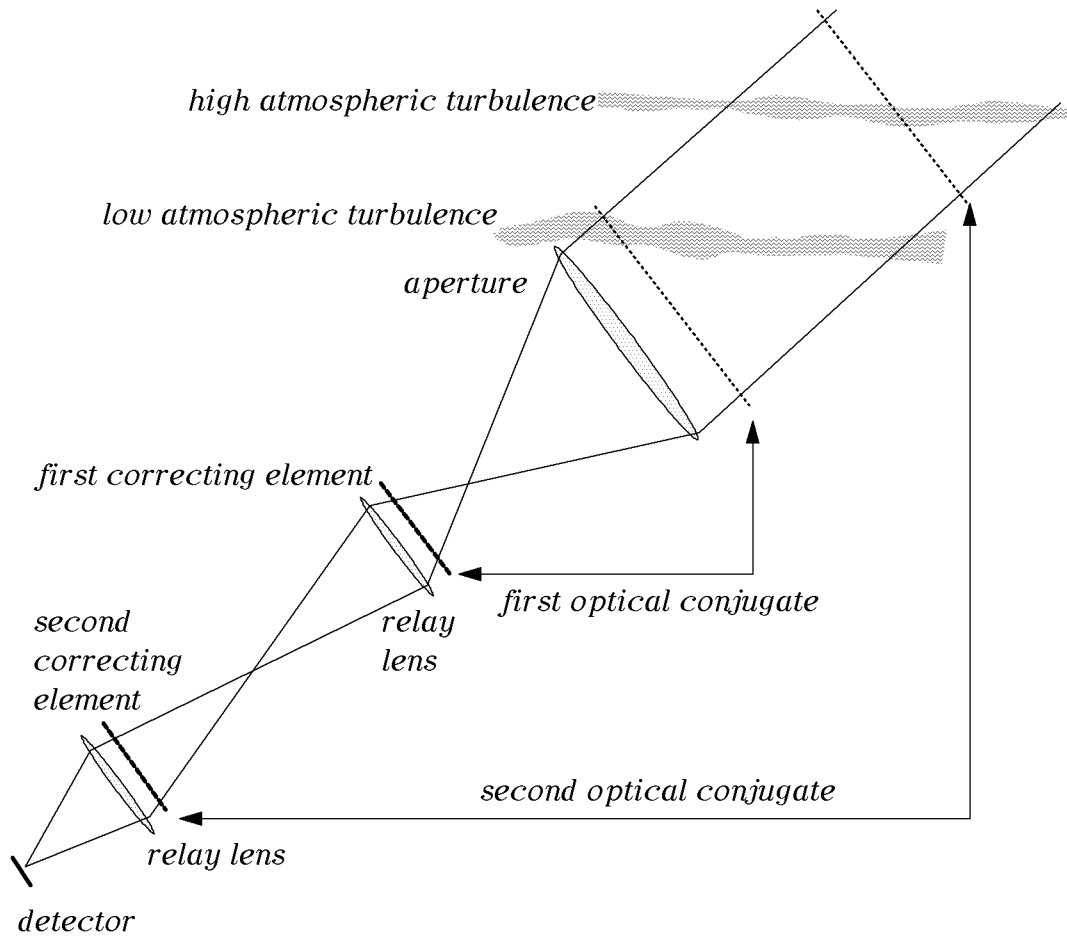


Figure 9. When the observed star is far from zenith, the atmospheric layers are tilted with respect to the optical axis of the telescope. As a result, the mirrors are no longer conjugate to their respective layers at all points.

images acquired by adaptive optics, and by deconvolution from a second wave front sensor [Roggemann et al. 1995] or from phase diversity [Love *et al.* 1996]. However, the huge advantages of adaptive optics, the long integration time and wide spectral band, will have to be compromised. The number of degrees of freedom for each layer is approximately twice the number of correlation cells at that layer (at the Nyquist frequency), or $(2D_i/r_i)^2$, where D_i is the projection of the aperture on the i th layer and r_i the corresponding r_o . For I layers we get

$$F = \sum_{i=1}^I \left(\frac{2D_i}{r_i} \right)^2 \quad (4)$$

Thus it seems that a better scheme should also include reduction in the number of pixels of the wave front sensors to match this number of degrees of freedom. These pixels must be sampled at twice the corresponding frequency for that layer, $2v_i/r_i$, where v_i is the local wind speed. The total bandwidth will thus be

$$B = \sum_{i=1}^I \frac{8D_i^2 v_i}{r_i^3} \quad (5)$$

A very small number of other parameters need be measured (the height, wind speed and correlation length for each layer), and those at a lower rate.

Another issue that needs to be looked into is the quality and sufficiency of laser guide stars. It is not easy to duplicate these lasers and other schemes require either too many laser spots or a fringe pattern at very high power, in contrast to the small number of degrees of freedom described above. Simpler schemes that would allow less powerful lasers should be sought.

A severe problem with realization of the multiconjugate systems is their optical design. Wave front correctors are mostly made of mirrors, which means folding the optical path as well as losing light. Refractive elements like liquid crystals are not available yet at the required quality. Thus, more effort should be invested to provide a better solution to the problem.

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