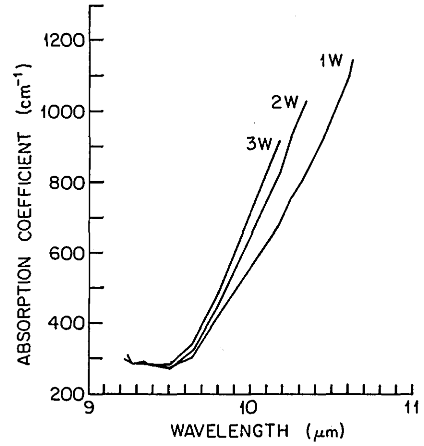

Fig. 1. Reflectivity as a function of wavelength for hot-pressed SiC with a polished surface at three different irradiation power levels.

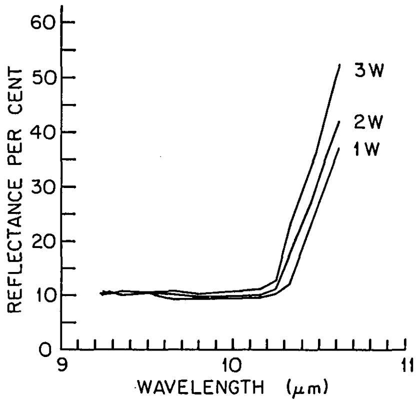

Fig. 2. Absorption coefficient as a function of wavelength for hot-pressed SiC at three different irradiation power levels.

the reflectivity is low and the absorption coefficient large. At the commonly used 10.6-μm line, the reflectivity is large and laser energy coupling efficiency is poor. To both optimize the energy coupling and minimize the number of parameters necessary to theoretically calculate substrate temperatures,[4,8] the 9.27-μm wavelength is more advantageous since the temperature dependence of the optical properties at this wavelength is negligible.

This work was supported in part by the Department of Energy under the technical supervision of R. J. Gottschall.

References

1. F. Shaapur and S. D. Allen, "Growth of Periodic Structures in Pyrolytic Laser-Deposited SiC," J. Appl. Phys. **60**, 470 (1986).
2. S. M. Copley, "Laser Applications," in *High Speed Machining Technology*, R. I. King, Ed. (Chapman & Hall, New York, 1985).
3. G. Q. Weaver and B. A. Olson, "High Strength SiC for Use in Severe Environments," in *Proceedings, Third International Conference on SiC* (U. South Carolina Press, Columbia, 1973).
4. M. Lax, "Temperature Rise Induced by a Laser Beam. II: The Nonlinear Case," Appl. Phys. Lett. **33**, 786 (1978).
5. W. G. Spitzer, D. Kleinman, and D. Walsh, "Infrared Properties of Hexagonal SiC," Phys. Rev. **113**, 127 (1959).
6. Samples were prepared by a Buehler Isomet Grinder/Polisher.
7. R. S. Browning, S. E. Wiberley, and F. C. Nachod, "Application of Infrared Spectrophotometry to Quantitative Analysis in the Solid Phase," Anal. Chem. **27**, 7 (1955).
8. F. Shaapur and S. D. Allen, unpublished results.

Phase closure with a rotational shear interferometer

Erez Ribak

Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, California 91109.
Received 13 September 1986.
0003-6935/87/020197-03$02.00/0.

Phase closure is a radio astronomical technique that enables recovering Fourier transform phases that would otherwise be corrupted by atmospheric and instrumental errors.[1] It has been suggested in the past that radio methods, such as aperture synthesis and phase closure, are applicable to the visible region too.[2,3] Recently direct phase closure was demonstrated on an optical telescope,[4] and it was shown[5] that the method of triple correlation[6] is a generalization of phase closure. Proposed here is a simple and efficient way of achieving phase closure by means of rotational shear interferometery.

In a rotational shear interferometer one images the telescope aperture onto the interferometer, then interferes the aperture with itself in a rotated orientation. If we observe an object $o(\mathbf{x})$ whose Fourier transform is $O(\mathbf{u})$, it can be shown[7] that the intensity of the interference pattern behaves like

$$I(\mathbf{u}) = O(0) + \mathrm{Re}[O(2\mathbf{u}|\sin\theta/2|/\lambda)]\,, \qquad (1)$$

where θ is the rotation angle and λ is the wavelength. It is obvious that to achieve the maximum frequency content permitted by the telescope the shear has to be 180°, but better dynamic range and SNR are possible for lower frequencies. This zoom effect is achieved by turning to smaller rotation angles. The second term on the right can also be written as

$$\mathrm{Re}[O(2\mathbf{u}|\sin\theta/2|/\lambda)] = |G(\mathbf{r})| \cos[\psi(\mathbf{r}) + \phi(\mathbf{r})]\,, \qquad (2)$$

where $\mathbf{r} = 2\mathbf{u}|\sin\theta/2|/\lambda$, $G(\mathbf{r})$ is the mutual coherence function, and $\psi(\mathbf{r}) = \arg[G(\mathbf{r})]$. $\phi(\mathbf{r})$ is the combination of all other phase fluctuations not associated with the mutual coherence function, mainly atmospheric but also those due to telescope imperfections etc. (No absorption in the atmosphere is assumed.) In phase closure one chooses three vectors $\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3$ so that $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = 0$. In this case the atmospheric and instrumental phases must obey $\phi(\mathbf{r}_1) + \phi(\mathbf{r}_2) + \phi(\mathbf{r}_3) = 0$. When adding the total phases for the three vectors, the atmospheric and telescope-induced phases cancel out, and we remain with the sum of the phases of the coherence function $\psi(\mathbf{r}_1) + \psi(\mathbf{r}_2) + \psi(\mathbf{r}_3)$. The same theory applies also to any closed loop of vectors, not just three.

If we choose a rotation angle $\theta = 120°$, we automatically create an infinite number of closure loops. Every equilateral triangle whose center coincides with the center of rotation will constitute such a set of vectors. At each vertex of such a triangle, we have an interference of two out of three possible points 120° apart on the telescope aperture [Fig. 1(a)]. The same applies for a rotation angle of 72° where we now have phase closure in all possible centered pentagons [Fig. 1(b)] etc. As remarked above, we do lose in terms of spatial frequency as we rotate at smaller angles. On the other hand, we could rotate by 144° and measure along the five-pointed star that is created as we connect all vectors 144° apart [Fig. 1(c)]. In general, for n vertices (or apertures, as referred to in radio astronomy) we have

$$n\theta = 2\pi m, \qquad m = 1,2,3,\ldots. \qquad (3)$$

Since the rotation angle θ is always less than π, we get $n > 2$ m

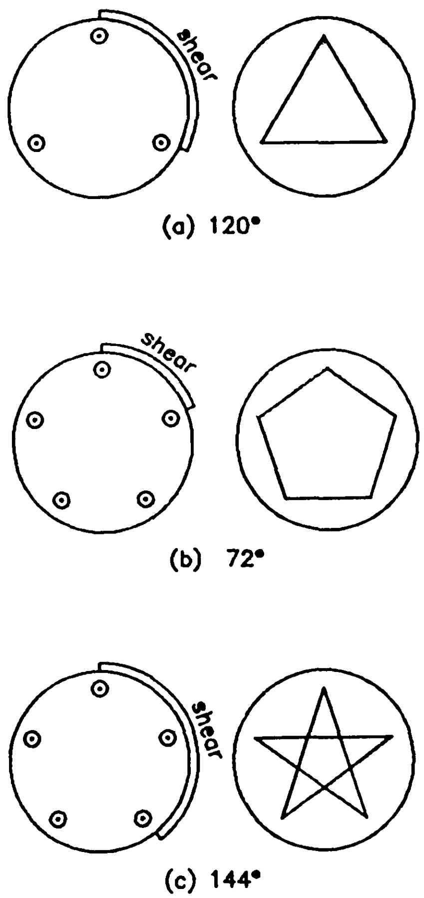

Fig. 1. (a) Two interfering apertures at 120° shear and one set of base lines in the telescope aperture that constitutes a closure loop. (All centered equilateral triangles are such loops.) (b) As in (a) with 72° rotation and pentagon. (c) As in (a) with 144° rotation and five-pointed star.

Table I. Possible Shear Angles and Associated Maximum Frequency (In Terms of the Telescope Cutoff Frequency f_c)

n (vertices)		3	4	5	6	7
$m = 1$	θ	120°	90°	72°	60°	52°
	f_{max}/f_c	0.87	0.77	0.58	0.50	0.44
$m = 2$	θ			144°	120°	103°
	f_{max}/f_c			0.95	0.87	0.78
$m = 3$	θ					154°
	f_{max}/f_c					0.97

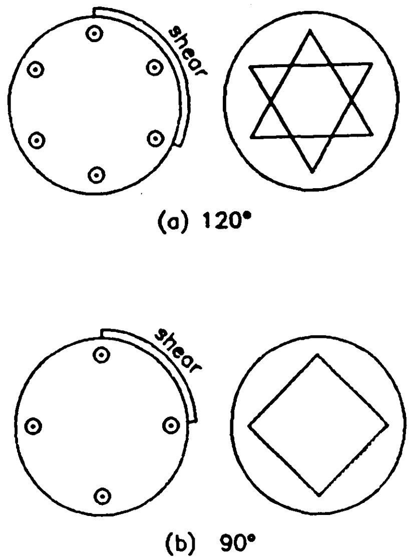

Fig. 2. As in Fig. 1(a) showing two redundant triangles with the same phase closure value. (b) As in Fig. 1(a) with 90° rotation. The sum of phases around the square (or any even-sided regular polygon) should always be zero.

(Table I). The loss of phase information at high frequencies, as inferred from this table, is not so severe. (Amplitudes can still be measured at 180° or up to the highest frequency.) This is because many methods have been developed to estimate the missing phases even from data that hold no phase information at all (Fienup, Knox-Thompson, speckle imaging, self-calibration, and many others[1,8–12]). These methods have shown their ability both in the visible and radio regimes.

A shortcoming of the proposed method is that for filled apertures, such as used in astronomy in the optical region, the number of closure loops is not sufficient to determine uniquely all the Fourier plane phases. In radio astronomy, where the apertures are few and (sometimes) far apart, the number of relations (aperture pairs) outnumbers the number of variables (coherence phases and amplitudes). Since the problem is overdetermined, methods like self-calibration, also known as hybrid mapping, are being used. In the interferogram of the two mutually rotated apertures, every base line (and thus every closure loop) exists twice [Fig. 2(a)]. This is due to the fact that the object $o(\mathbf{x})$ is real, and its Fourier transform $O(\mathbf{u})$ must be Hermitian. If the interferogram emcompasses N pixels on the (2-D) detector, we will have N/n phase closure relations, out of which one-half is equal (up to the noise) and opposite in sign to the other half. Thus there is not enough redundancy to solve for all the needed phase relations. This is the case for odd values of n, whereas for even values of n each closed loop has equal and opposite sides, and the coherence function phases are equal and opposite in sign [Fig. 2(b)]. Included in this category is the $n = 2$ case (180° rotation) not mentioned in the table. Since the closure phase is trivially zero in even regular polygons, they can only be used to enhance the signal-to-noise of the amplitude measurements.

A drawback of the scheme compared to radio astronomy is that all the base lines have to be of the same length, as set by the shear vector $\mathbf{r}$. Every point in the interferogram is involved in only one phase closure loop. This base line limitation also rules out amplitude closure, which is another tool in radio astronomical interferometry.[1] Amplitude closure cannot be applied directly, since it demands that measurements be taken simultaneously over base lines of different lengths. One should also keep in mind that the quantity measured is the local intensity, and the phase value related to this intensity has still to be extricated.[3] This is especially difficult under low light level conditions or near zeroes of the mutual coherence function. On the other hand, once we have phase closure values for one interferogram, we can average over many interferograms to improve the SNR.

There are different approaches as to how to collect and use the phase closure data. One way is to take measurements at lower shear angles and try to build a partial phase and amplitude map consistent with these data and other physical demands, such as source positivity and limited size. The shear value is raised, as is the frequency content of the data, and a map utilizing the former map and the new data is constructed. The process is repeated until the highest frequency possible is attained. The other way round is also possible: start at a high frequency, high rotation angle, then interpolate between points by going to a simple fraction of the first angle, etc. Again, we demand that in every step we have consistency with object positivity and limited support. A third way is to build the phase map from one shear value by demanding that the phases are continuous as one proceeds from one polygon to the adjacent one (both in the radial and tangential directions). This approach is equivalent to the maximum-entropy demand of smoothness, this time in Fourier space. Notice also that if one employs maximum entropy methods, phase closure values from interferograms of different shear can be combined as constraints. The increase in number of phase relations improves the quality of the solution. Of course, there is always the approach of radio astronomy, where a model is iteratively improved on under the closure constraints.[1] A combination of different methods is also possible.

To summarize: phase closure can be realized on existing telescopes and existing interferometers without special modifications. Although not all base lines are possible, the extra constraints provided by the closure phases reduce very much the ambiguity now existing in phaseless image reconstruction.

The work described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. The author is on leave from the Optical Sciences Center of the University of Arizona.

References

1. T. J. Pearson and A. C. S. Readhead, "Image Formation by Self-calibration in Radio Astronomy," Ann. Rev. Astron. Astrophys. **22,** 97 (1984).
2. W. T. Rhodes and J. W. Goodman, "Interferometric Technique for Recording and Restoring Images Degraded by Unknown Aberrations," J. Opt. Soc. Am **63,** 647 (1973).
3. D. H. Rogstad, "A Technique for Measuring Visibility Phase with an Optical Interferometer in the Presence of Atmospheric Seeing," Appl. Opt. **7,** 585 (1968).
4. J. E. Baldwin, C. A. Haniff, C. D. Mackay, and P. J. Warner, "Closure Phase in High-Resolution Optical Imaging," Nature London **320,** 595 (1985).
5. F. Roddier, "Triple Correlation as a Phase Closure Technique," Opt. Commun. **60,** 145 (1986).
6. A. W. Lohmann, G. P. Weigelt, and B. Wirnitzer, "Speckle Masking in Astronomy: Triple Correlation Theory and Applications," Appl. Opt. **22,** 4028 (1983).
7. F. Roddier, "Rotation Shearing Interferometry," in *High Angular Resolution Stellar Interferometry* (IAU Colloquium 50) J. Davis and W. J. Tango, Eds. (Astronomy Department, School of Physics, U. Sydney, 1979).
8. R. H. T. Bates, "Astronomical Speckle Imaging," Phys. Rep. **90,** 203 (1982).
9. F. R. Fienup, "Reconstruction of an Object from the Modulus of its Fourier Transform," Opt. Lett. **3,** 27 (1978).
10. K. T. Knox and B. J. Thompson, "Recovery of Images from Atmospherically Degraded Short Exposure Photographs," Astrophys. J. Lett. **193,** L45 (1974).
11. A. Chelli and J. M. Mariotti, "Visibility and Phase Analysis for Image and Pupil Plane Interferometry at Optical Wavelengths," Aston. Astrophys. **157,** 372 (1986).
12. E. Ribak "Astronomical Imaging by Filtered, Weighted Shift-and-Add," J. Opt. Soc. Am. A **3,** 2069 (1986).

Eigenstructure approach to directions-of-arrival estimation in IR detector arrays

Daniel Spielman, A. Paulraj, and Thomas Kailath

Stanford University, Information Systems Laboratory, Stanford, California 94305.
Received 14 June 1986.
0003-6935/37/020199-04$02.00/0.

Eigenstructure (also called subspace) methods were first introduced by Pisarenko[1] in time series analysis for extracting harmonics embedded in white noise. They were generalized by Schmidt[2] and Bienvenu and Kopp[3] who applied them to the directions-of-arrival (DOA) estimation problem. Their chief advantages over conventional methods are that they produce estimates possessing apparently higher resolution and yield asymptotically exact estimates for the DOAs. The applications of these methods have so far been confined to the so-called coherent data models, i.e., where the sensors measure amplitude and relative phase or equivalently, in the narrowband case, the familiar complex envelope. We consider the extension of such eigenstructure or subspace techniques to situations where incoherent or power measurements from IR detectors are available. Such detectors may be of the thermal or photon type and produce an output voltage proportional to the total power incident on the detector. We propose a subspace algorithm for high-resolution estimation of IR emitter directions in which the estimates are obtained as intersections between the so called signal subspace and an array manifold. Simulation results are presented that substantiate the performance of our method.

Consider an array of arbitrary (for example, conformal) geometry consisting of m IR detectors. The frequency spectral responses of the detectors are assumed to be identical. Each detector has an associated condenser lens (a simple lenslet would do) so that the detector receives the incident IR radiation with a directionally selective (although again arbitrary) response determined by the lenslet parameters and the angular orientation of the detector assembly. We assume that no two-detector assemblies have the same absolute angular response or differ from each other by a scalar. Such a differential angular response could be obtained even with identical but directional detectors if their individual maximum response axes are oriented differently; see Fig. 1.

Let there be $d < m$ mutually uncorrelated broad spectrum IR sources sufficiently far from the array to allow a planar wavefront approximation and let each source have a distinct although unknown frequency spectrum that may in fact be possibly time varying. The source directions are assumed to be constant over the entire measurement period. A diffuse noise field (such as sky noise) that has known angular and frequency spectral characteristics is present. We assume that there is a selectable spectral shaping filter built into the array either locally into the detectors or incorporated into the overall IR window that may cover the array so that it is possible to weight differentially the frequency spectra of the incident radiation before it reaches the detectors. These frequency shaping filters need not necessarily be narrow-