

Measuring turbulence in an extended field of view

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ABSTRACT

Adaptive optics is limited today to correction of turbulence inside a cone extending from the reference source to the telescope aperture. Even when the reference source is a natural star, the measured – and corrected - cylinder does not allow observation of most extended astronomical objects. Hence the search is on for a method to measure turbulence in a conical volume which opens up from the telescope upwards. Various schemes were proposed to widen the field of view by using more artificial or natural guide stars, and by processing the measured data in different ways. It has been shown experimentally that the existence of three natural stars around the rim of the required cone is sufficient. Using multiple laser guide stars, schemes varying from separation of measured volumes and stitching of their edges, to integrated methods were suggested. It was also proposed to infer the turbulence from the shape of the beams as they propagate up in the atmosphere. Structured light above the turbulence is another option that was raised. Such a grid is created by interference of laser beams or by interference of powerful radio beams that break down the air into visible plasma. It can be shown that these fringes, either from a laser or from radio, can be analyzed optically, reducing the power requirements significantly. This field of atmospheric tomography is likely to produce soon corrected images of extended astronomical objects. In addition, being able to separate the contribution of the atmospheric layers, we will acquire better knowledge of atmospheric turbulence.

Keywords: Adaptive optics, multi-conjugate adaptive optics, laser guide stars, atmospheric tomography.

1. INTRODUCTION

Standard adaptive optics systems measure and correct the wave front error along the path to the reference star. Because this path is very narrow, it also limits the width of the correction. The problem is even more severe when the reference star is not the measured star: either it is another natural star which could be rather far away or an artificial guide star at tens of kilometers above the telescope. In both cases the measured volume of atmosphere (Figure 1) does not overlap with the observation path to the star and its vicinity, namely a truncated cone whose bases are the telescope and the star field. Even for bright objects, the corrected isoplanatic angle extends only over a few arc seconds. One would like to have wider fields, of arc minutes and more, over which turbulence can be measured and corrected.

Wider field of view can be achieved by multi-conjugate adaptive optics. This method deals with measurement of atmospheric turbulence at various elevations and its correction by using more than one optical element, usually conjugate to the most offending layers. The idea was proposed very early on by Dicke¹ and by McCall and Passner² and became more relevant with the suggestion of laser guide stars in the 1980s³⁻⁵. However, in the same manner that technical difficulties delayed the application of adaptive optics in general, multi-conjugate adaptive optics is slow in application, and only three experiments were performed in this direction⁶⁻⁸. Like the rest of adaptive optics, there are some issues that will have to be addressed before the method is applied in full.

2. SEPARATION OF THE ATMOSPHERE INTO THIN LAYERS

In adaptive optics systems we have a correcting element, such as a flexible or segmented mirror, or a liquid crystal phase modulator. In order to have maximum effect these mirrors have to be conjugated to the most offending atmospheric layers, or be placed strategically, so as to maximize their impact on the performance of the system. Because wave front measurements are usually photon-limited and noisy, we wish the sub-aperture size to expand so as to collect more light. However, it is limited by the size of r_0 for each layer. Thus, knowledge of the actual r_0 of the separate layers is also useful, as well as the wind speed and direction in the various layers. Is it at all justified to assume that the atmosphere is separable into thin layers? Various statistical measurements were developed to measure the turbulence ‘constant’ $C_n^2(h)$ as a function of altitude h . Data

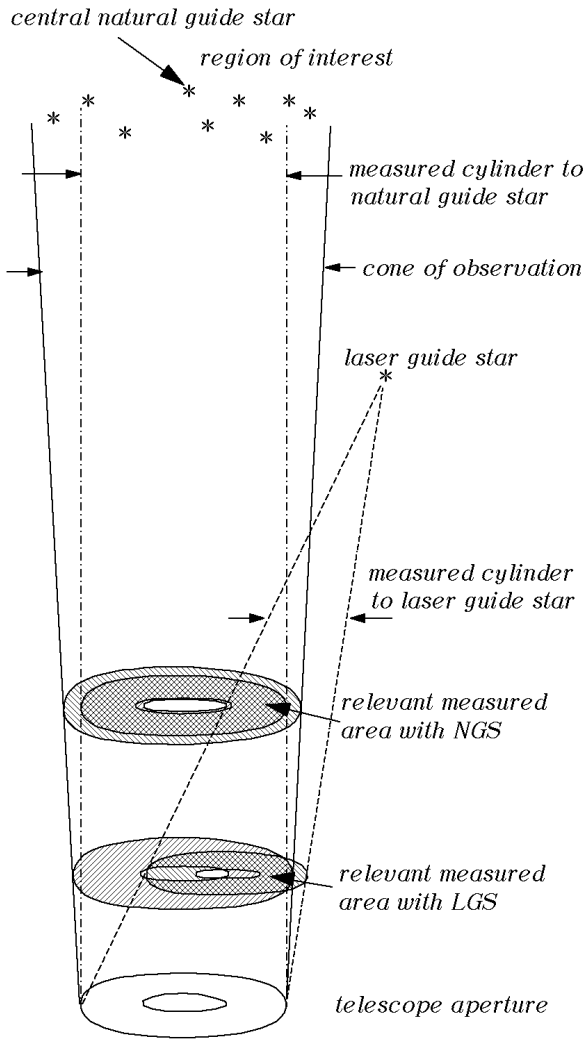


Figure 1. Light shading represents the area we wish to measure for an extended field of view. Darker shading (top layer) signifies area measured with a natural guide star, and (bottom layer) with a laser guide star.

the wave fronts at a rate faster than the atmospheric changes, for the corrections to be effective.

3. MEASUREMENTS OF THE VARIOUS LAYERS. NATURAL AND ARTIFICIAL GUIDE STARS.

Fast wave front sensing is crucial for adaptive optics, and even more so for multi-conjugate adaptive optics. A multitude of methods has been proposed, and new ones are still required. Once it is grasped that atmospheric tomography is the issue at hand, it also becomes apparent that a diversity of sources and a diversity of detectors is required to solve the problem.

Dicke¹ was the first to suggest of multi-conjugate adaptive optics, involving only a single star (Fig. 2). Making use of a phase contrast wave front sensor, alternating corrections at two mirrors were shown to be enough to correct the cylinder of turbulence between the telescope and the star¹⁴. Even this simple approach shows the attraction of multi-conjugate adaptive

collected at many telescope sites indicate that at very good sites the number of layers is rather small (1-2), and it becomes large for worse sites⁹. Henceforth we shall assume that indeed the atmosphere is made up of few thin layers.

To achieve a wide field of view one must measure the instantaneous values that wave fronts take as they propagate down the atmosphere, along and near the optical axis of the telescope. In essence, this is a tomographic measurement of the refractive index of air along the relevant path of light. In the astronomical case this tomography becomes both simpler and more complex. Simpler because turbulence layers tend to lie in horizontal planes; usually there is no need to study the full three-dimensional structure. Instead, only a limited set of parallel planes have to be measured, perhaps only one or two significant layers. Since the statistics of turbulence usually obey Kolmogorov's law, each turbulence layer need be measured only up to a resolution of $r_0/2$ of that layer (r_0 is Fried's parameter signifying a region of rather constant wave front phase). The same applies to the temporal resolution - one can integrate up to $(r_0/v_{\text{wind}})/2$ of each layer, where v_{wind} is the wind speed in that layer. Again, because of Kolmogorov's spectrum, unruly solutions can be easily voted against. Alternatively, constraining the solutions to Kolmogorov's limitations may make the calculation faster. Because of Taylor's hypothesis of frozen flow, previous measurements of the turbulence layer provide us with some prior knowledge of the current shape of the wave front¹⁰⁻¹².

On the other hand, tomography is made more difficult because the amount of light available is usually not sufficient. Natural and artificial guide stars provide too little flux for wave front measurements to be accomplished without excessive noise. In addition, the distribution of available light sources is random, too sparse, and arbitrary if one uses only natural stars, or not high enough, if laser guide stars are employed. If laser power is limited, then there might be fewer beacons than required. The sensors are all at the bottom of the telescope, and can sample only that section of space visible from this location. The lasers also emanate from the vicinity of the telescope. The sensors and the lasers can be distributed in nearby locations, but this is usually done for measurement of the global tilt of laser guide stars¹³. This limits very much the variability required for tomography. The measurements must also be solved to provide

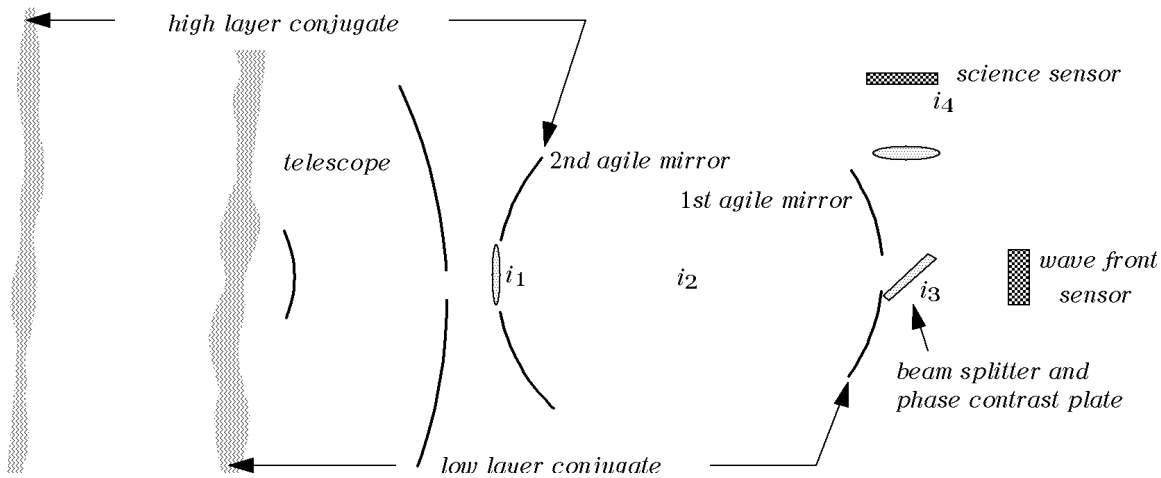


Figure 2. In this prototype¹ multi-conjugate adaptive optics, the telescope, first and second deformable mirrors and beam splitter create images i_1 - i_4 . Measurement is done by phase contrast alternatively on the atmospheric layers.

optics: suppose that only the layer near the telescope (boundary layer) is corrected successfully. Then all other sources in the field will now be affected only by higher turbulence, usually less severe than the bottom one. Their correction can be achieved by locking on other stars, further off in the field. This is because all the stars in the field of view share the bottom layer turbulence. Alternatively, correcting the higher turbulence alone results in larger isoplanatic angles.

The suggestion made by Hudgin in 1980 and by Feinlieb in 1981 to use laser guide stars for military purposes³, and a similar but independent proposal for astronomy⁴ made the guide star problem simpler: there was no need any more to rely on the observed star itself as the reference beacon. It was grasped from the start^{4,15-18} that many sources could be hung in the sky, and their geometrical distribution was in the hands of the user, limited only by technical constraints (Fig. 3). Two kinds of sources were proposed: Rayleigh guide stars and sodium guide stars. The first kind uses light scattered by dust in the lower atmosphere (<40km), usually from a ultra-violet to visible laser. The second kind is tuned to the sodium *D* line at 589nm, and is scattered from a sodium layer at elevations of 88-95km. It seems that a web of guide stars, Rayleigh or sodium ones, are sufficient to solve the tomographic problem successfully^{4,15-23}. However, it was found that a smart approach will have to be taken, and even then the system might not be effective if the atmosphere cannot be modeled by thin layers alone²⁴.

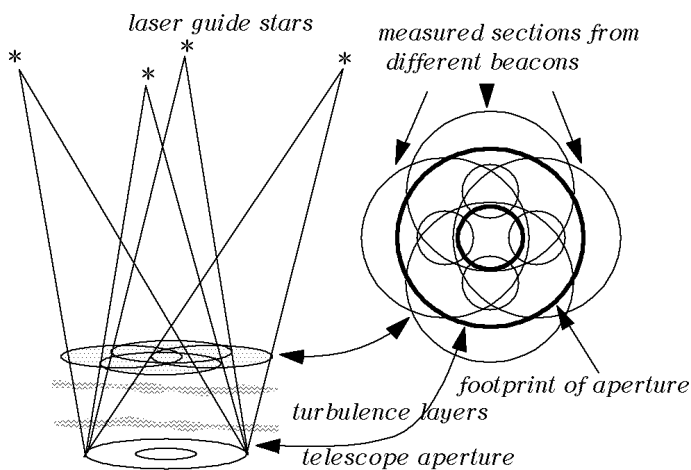


Figure 3. It takes at least three laser guide stars, around the rim of the field of view, to sample adequately the turbulence layers. In the case of four lasers, as shown here, redundancy has to be weighted against the price of the extra laser.

If one wishes to have a wider isoplanatic patch, and is willing to give up on very accurate correction, then a Rayleigh beacon might be sufficient. This is because such a beacon samples the lower atmosphere only, and that section of the atmosphere is responsible for most of the errors common to the wider field of view^{21,22}. This argument was proved by measurements²⁵.

The number of sodium guide stars must be equal or greater than the number of thin atmospheric layers^{15,26,27}. Using arguments of isoplanaticity with a continuous (non-layered) atmosphere, one arrives at similar numbers for sodium stars and much higher numbers (in the hundreds) for Rayleigh stars^{16,28}.

Approaches using Rayleigh stars for solution of the lower layers and sodium stars for the higher layers^{18,19,21,22,29} were shown to increase the field of view. However, the residuals from the Rayleigh measurement are comparable to the sodium

measurement alone, and might make the Rayleigh beacons redundant²⁹. If the number of beacons is low (three to four^{15,16,21,22}), then they should be placed around the rim of the telescope field of view (Fig. 3). In these studies, Rayleigh stars were placed under the corresponding sodium stars. Still, there is no complete study that compares the various arrangements. These arrangements depend strongly on the location of the wave front sensors, what volume of space they measure, and whether this volume is shared with the other sensors or not.

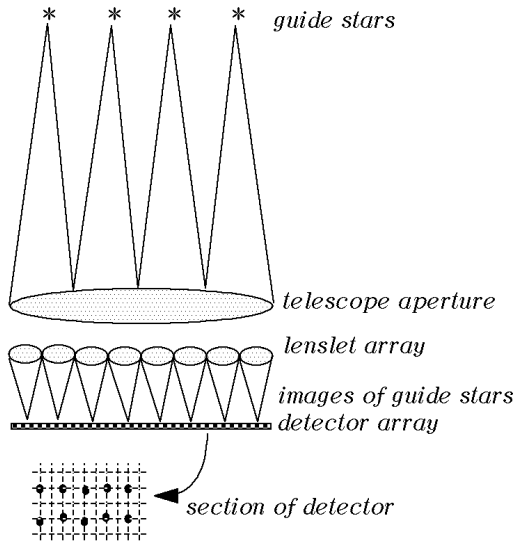


Figure 4. When stitching different measurements of the atmosphere, each section of the telescope aperture views only one beacon. The data from the different sections are then combined.

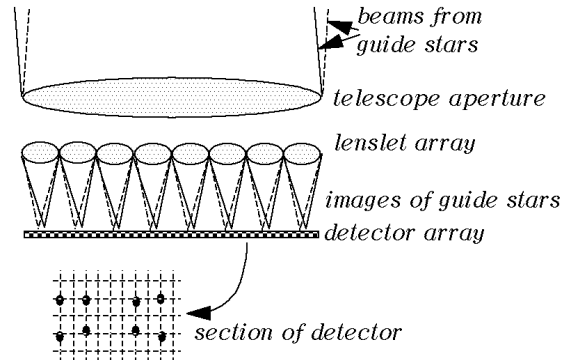


Figure 5. It is more efficient to have each lenslet observe more than one guide star. This way the pixels in the detector are used more efficiently.

The placement of the wave front sensors is different for the different schemes. One approach is to make each sensor measure a wave front from each laser guide star, through a section of the telescope aperture (Fig. 4). This samples the turbulence in nearby sections, which are then combined together in a process dubbed butting or stitching^{25,30}. This is not a very efficient process, because the separate patches have different tilts which cannot be measured^{31,32}. Tyler²¹ literally widens the basis of the earlier approaches. He shows the benefits of having each sensor measure the full cone from each star to the whole aperture; sharing the lower part of the atmosphere helps measure the tomography of the atmosphere. Part or the whole of the aperture are used for sending up the laser beams as well as for sensing them through parts or over the full aperture⁷.

The method proposed now is to use the Hartmann-Shack sensor to sample one star in each lenslet, with a number of pixels on the CCD. This way sampling is more robust than when sampling with the minimum of four pixels. However, if more than one star hits that section of the CCD (Fig. 5), the efficiency (stars/pixels) is even higher.

How to extract the phases of the atmospheric layers out of the tomographic measurements? The approach proposed initially was to shift and average the wave front sensor measurements as they propagated down the turbulence through different paths⁵. This is justified since the phase differences between the various measurements are rather small^{15,19,27}. A set of equations can be set up for each guide star and for each layer, with the appropriate shifts for the different layers. Suppose there are K turbulence layers at elevations $h(k)$ $\{k = 1..K\}$, and the guide star is at altitude H . The phase at layer k and coordinate \mathbf{r} is $\mathbf{y}(k, \mathbf{r})$, and its propagation to the ground creates a phase $\mathbf{j}(k, \mathbf{r})$ which is actually a convolution with a harmonic function³³ $\mathbf{h}[h(k, \mathbf{r})] = \mathbf{h}(k, \mathbf{r})$:

$$\mathbf{j}(k, \mathbf{r}) = \mathbf{y}(k, \mathbf{r}) * \mathbf{h}(k, \mathbf{r}).$$

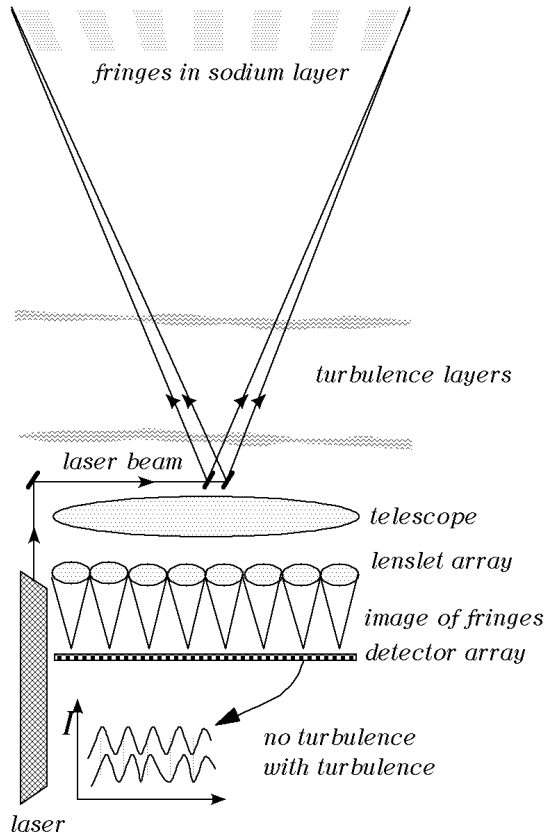


Figure 6. Using a high-power laser, it is possible to create fringes in the extended field of view. Each lenslet in the sensor images many such fringes. General movement of the fringes signifies turbulence in the lower layer, and their distortions – turbulence in the upper layer. These are calculated and integrated.

Because of geometric effects, one measures a linear combination $G(m, k)$ (essentially scaling and shifting) of the elements of $\mathbf{j}(k, \mathbf{r})$ which depends on the location of the m th guide star

$$\mathbf{j}(m, k, \mathbf{r}) = G(m, k) \mathbf{j}(k, \mathbf{r}) \{1 - [h(k)/H]\}^w,$$

where the last factor arises because wave front sensing measures the w th derivative of the phase (the phase itself, its gradient, or its curvature; $w=0,1,2$). The measurements for all the guide stars are

$$\mathbf{M}(m, \mathbf{r}) = \sum_k G(m, k) \mathbf{j}(k, \mathbf{r}) \{1 - [h(k)/H]\}^w$$

These equations have to be inverted for $\mathbf{j}(k, \mathbf{r})$ and deconvolved to yield $\mathbf{y}(k, \mathbf{r})$, the required phases. If the small effect of propagation is ignored, these equations can be solved by least-squares fitting³⁴.

Sandler¹⁷ proposed to use an ordered array of beacon laser spots spaced so as to have matching fringes in the shearing interferometer wave front sensor. The results of the experiment were inconclusive, but the idea brought about an opposite idea: using beacon laser fringes over tens of meters to match a set of Hartmann-Shack sensors. This is now a means to separate lower and higher turbulence layers²³. The projected aperture of the telescope is broken into a number of Hartmann-Shack sensors, each facing most of the fringe pattern in the sky. As in the standard sensors, the whole fringe pattern will shift with slope errors, mostly contributed by the low-lying turbulence. At the same time, the images of the fringes will suffer from distortion from high turbulence. Thus the phase errors inside each sub-aperture can be traced, and added up with the neighboring sub-apertures to yield the high turbulence. Unfortunately, after detection this addition is incoherent and thus inefficient (see below).

The main disadvantage of the multiple-beacon schemes is their requirement for high power lasers. Creation of many point beacons merely multiplies the requirements from a single beacon, and makes the projection system cumbersome. Either three to four lasers will have to be employed in parallel, each with required power of approximately 8-15 Watts. Alternatively, the beam will have to be scanned across the sky (multiplexed stars) at high rate. Using very conservative calculations, the fringe method requires up to 500 Watts of laser power, but it has a rather simple projection system, that of a simple interferometer (Fig. 6). In addition, it does not saturate the absorption of the sodium layer, as might happen with tight beacon spots.

Phase-diversity methods³⁵ were used on solar features to solve for the wave front and separate low from high aberrations. Although not fully successful, their results are very encouraging. They show that even low-contrast features, which evolve in time, are sufficient to tell apart the different layers. They also show the validity of the method, making use of a simple measurement, which only requires one sensor at focus and one sensor out-of-focus.

Another result of their research is that there might not be a special need for a spatial arrangement of laser stars or laser fringes. The sodium layer and the light scattered off the sodium are very inhomogeneous. Simple sodium lamps (such as those used for street lighting³⁶ or miniature ones³⁷) might be sufficient to illuminate a very large section of the sky. Laser light is much more collimated than incoherent light, but it might be possible to use non-imaging concentrators to get sufficient amount of light in the relevant area. The natural clumpiness of the sodium will result in inhomogeneous back

scattering, which could be sufficient for wave front sensing. If the returned intensity is still too smooth, radio beams could be employed to modify it. For example, a radio interferometer could modulate spatially the illuminated sodium layer and create fringes in the returned light³⁶.

Other means were proposed to measure the different layers. Curvature sensing relies on intensity variations down the beam from phase variations. But that effect also means that variations of the intensity at the aperture of the telescope, better known as scintillation, are related to phase variations at the high atmosphere, kilometers above the telescope. These intensities can be inverted to find the original phases^{38,39}. Intensity variations are measured anyway by wave front sensors and discarded as noise. Instead, they can be used for this purpose, especially for brighter sources and stronger turbulence.

The placement of the wave front sensor, at a plane conjugate to a specific layer, is significant. Fuchs et al.⁴⁰ proposed and tested in the lab the possibility of moving the wave front sensor up and down the optical train. They showed that it allows to cancel scintillation effects in the layer conjugate to the measured one. But another advantage that emanates from their work is that two curvature sensors, placed at the conjugates of the worst layers, will each measure the other layer simultaneously. Notice how similar this now becomes to Dicke's method¹.

4. OPTICAL DESIGN

How big a field of view can we expect with current technology? It so turns out that, like in many other cases, we might be limited more by the detectors than by other elements. Consider the following example: We have electronic cameras (charge-coupled-devices, for example) comprising $N = 8,192$ pixels (in one or a few tiled cameras). Suppose also that we have a $D = 5\text{m}$ telescope. At the wave length of $\lambda = 600\text{nm}$ and at maximum resolution, each pixel will see one-half the maximum angular resolution or $0.61\lambda/D = 0.0732$ microradians each. The whole field of view will be $8192 \cdot 0.0732 = 586$ microradians or two arc-minutes. Thus the total field of view is $0.61\lambda N/D$. Larger fields can thus be achieved only by lowering the resolution per pixel, using larger cameras or smaller telescopes. Wave front sensors that need to measure slightly beyond the edge of the projected telescope aperture might require somewhat larger fields of view.

While planning an adaptive optics system, certain issues arise that might become more complex with a multi-conjugate system. In some cases the multi-conjugate system is a simple extension of the single conjugate system with similar requirements (for example, that the whole system could be pulled out to allow for wide field imaging at low resolution without the benefit of adaptive optics). In other cases special issues will have to be answered, such as the quality of the images of the conjugate planes when relayed down the optical train⁴¹. The tendency to use reflective (rather than refractive) optics, brought about by the need for good infra red imaging, is very demanding. For this regime, off-axis paraboloids and hyperboloids have to be considered, with their severe limitations.

Other issues stem from the requirement for conjugation to various - and variable - atmospheric turbulence planes. Variations of the elevation of the turbulence layers over minutes⁴⁵ might require swift changes. Also, the varying size of r_0 requires zooming capabilities on top of the re-imaging capabilities (assuming that the deformable mirror has a fixed geometry with a limited number of elements). At the same time both the scientific detector and the wave front sensor or sensors should stay at the same location or be moved in unison (to within a fraction of a pixel!). The movement of the optical elements has to be designed so as not to have them interfere with each other's path⁴¹.

Most adaptive optics systems today tend to put the deformable mirror at a position conjugate to the aperture of the telescope. This is useful only in spectroscopic measurements, when one wishes to maximize the amount of energy in the slit⁴³. For imaging applications, a much better solution results when the deformable mirror is conjugated to the worst layer, usually higher up. The main disadvantage of this method is that there is vignetting leading to loss of light. This can be countered by employing a tomographic measurement of the turbulence and its correction outside the truncated cone from the telescope to the guide star(s) (Figure 3).

Because of vignetting, large areas of the corrected plane cannot be measured because of its poor overlap with the telescope aperture. The degree of overlap is further decreased when using a laser guide star. In this case not only is the area of the higher layer smaller than that of the aperture, but it is also scaled differently. Finally, there is also the problem of mismatch (both in registration and in scale) between the wave front sensor and deformable pixels when conjugated to different layers⁴⁴. It seems that a large number of these problems can be solved by oversampling the turbulence in four dimensions: beyond the

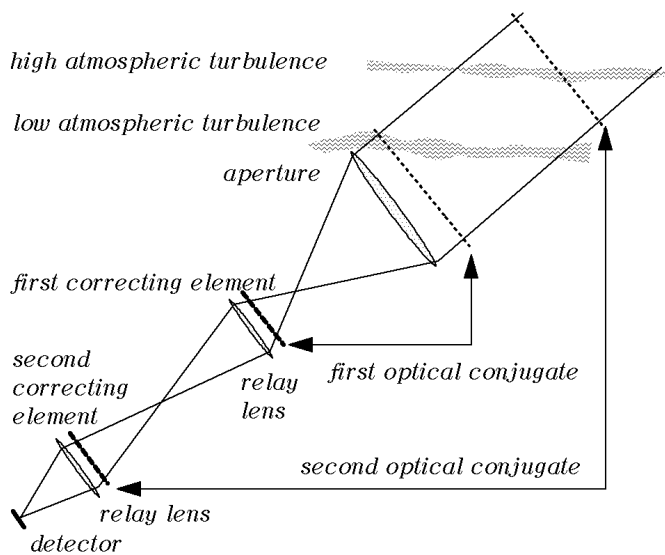


Figure 7. Off-zenith, both measurement and correction are not co-planar with the turbulent layers.

aperture, at different elevations, and at former time steps. Spatio-temporal prediction using the fractal nature of the wave fronts¹² should replace all but the highest missing frequencies and those upwind from the guide star(s).

Another issue is the order of the correcting elements. When turbulence is weak, the geometric approximation (that beams only bend, but do not diffract or cross) is valid, and the phase errors from the different layers add linearly³³. Thus they can be subtracted linearly without regard to which conjugate layer is corrected first. If turbulence is not so weak, it might be better to conjugate and correct first the lower atmospheric layers, and then re-image and correct the higher ones². In this manner the errors are undone in opposite direction to the order of their occurrence³⁴.

How to deal with objects that are not at zenith? In such a case, the atmospheric layers do not lie normal to the direction of observation, and a corresponding tilt to the sensors and the deformable mirrors is needed (Fig. 7). This formidable optical design problem might not have a proper solution, and might limit wide field adaptive optics to the vicinity of the zenith.

5. FUTURE DEVELOPMENTS

Multi-conjugate adaptive optics is considered as the next step after standard adaptive optics has proven its utility, reliability, and usefulness for the average astronomer. Only when prices drop and more experience and confidence is gained, will there be room for expansion to more laser guide stars, more sensors and more mirrors.

The number of degrees of freedom for each layer is approximately four times the number of r_0 cells at that layer (at the Nyquist frequency) sampled at twice the corresponding frequency for that layer. A very small number of other parameters need to be measured (the height, wind speed and r_0 for each layer), and those at a lower rate. Thus it seems that a better scheme should also include reduction in the number of pixels of the wave front sensors to match this number of degrees of freedom. This approach was recently raised as a result of the experimental success to show that wave fronts can be interpolated between neighboring guide stars⁸. In this experiment, pyramid wave front sensors were used, which split the stellar image between four detectors, similar to quadrant detectors in the Hartmann-Shack configuration. However, in the pyramid sensor, a whole atmospheric layer can be imaged on a contiguous detector. In direct continuation⁴⁵, two or more such layers can be imaged, each with its own pixel size, corresponding to its own r_0 . Moreover, one can now use – in parallel – many pyramids for many stars, combining their light on the layer detectors⁴⁵.

Another issue that needs to be looked into is the quality and sufficiency of laser guide stars. It is not easy to duplicate these lasers, and various schemes require either many beacons or a fringe pattern, all requiring high power, in contrast to the small number of degrees of freedom described above. Simpler schemes that would allow less powerful lasers should be sought.

One such scheme is combining laser fringes and pyramids⁴⁶. In this scheme one interferes laser beams at the sodium layer, creating a criss-cross of fringes whose period must be 1m or higher. The telescope images these fringes onto a corresponding array of pyramids, and each pyramid splits the fringe crest into four fields. Further down the line, we have four images of the top layer, each a combination from all parallel pyramids (Fig. 8). Subtraction of each two fields from the opposite ones provides the cartesian derivatives of the wave front at this layer⁴⁵. Even further down we have the four images of the next atmospheric layer, which can be split optically and detected separately (Fig. 9). The separate detection now allows the detectors to match r_0 for each layer. Hence, if a single layer requires approximately 12W of laser power, then two layers require less than twice that power, say 20W. This is a significant saving compared to the previous Hartmann-Shack detection scheme of laser fringes²³, where 500W were required. The saving of power is due to the optical addition of light, which is more efficient than their digital addition.

Another option raised recently is to use Rayleigh laser guide stars, and follow their shape as they traverse the turbulence⁴⁷. Choosing the wave-length to be in the ultra-violet allows significantly more returned power than sodium beacons. In this option, one sends dozen or more beams around the rim of the telescope, and few more from above its central obscuration. As the beams move up, they scatter from atmospheric dust up to 40km. If one focuses the telescope on a specific elevation, then the beams seem to be points or slightly elongated lines (diffraction is insignificant: their diameters are less than 10cm). Below and above that plane, they defocus into the aperture shape. Using these defocused images, one can apply phase diversity³⁵ and calculate the shape of the wave front.

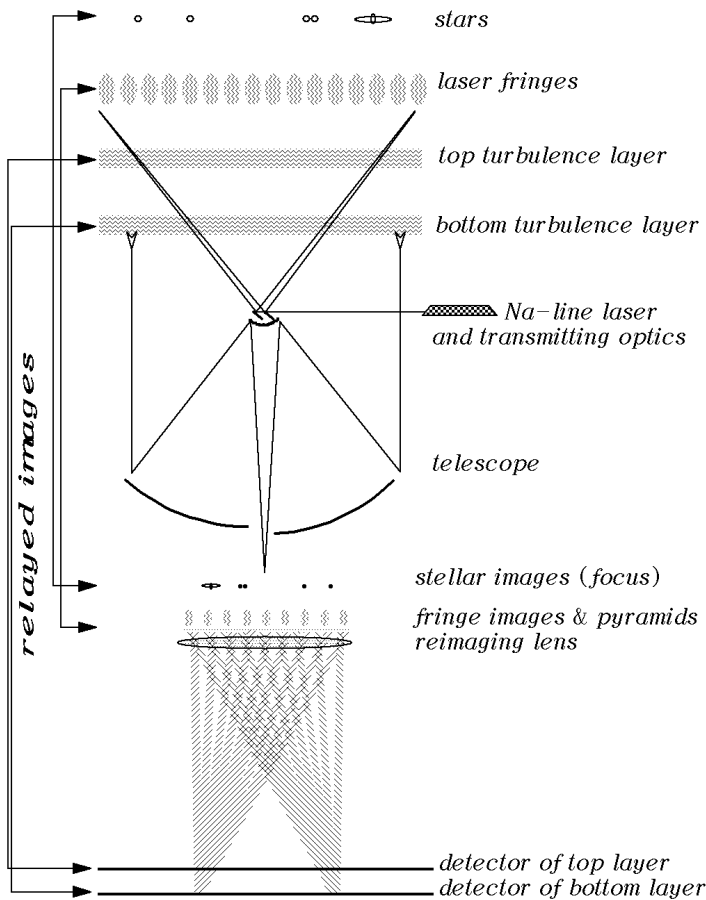


Figure 8. Significant reduction in laser power is achieved by optical processing. Instead of measuring turbulence and computing the contribution of the layers, the turbulence is measured directly and separately in the different layers. Each fringe is divided into four beams (two shown) at its center. The difference between the two images is proportional to the wave front derivative in that layer.

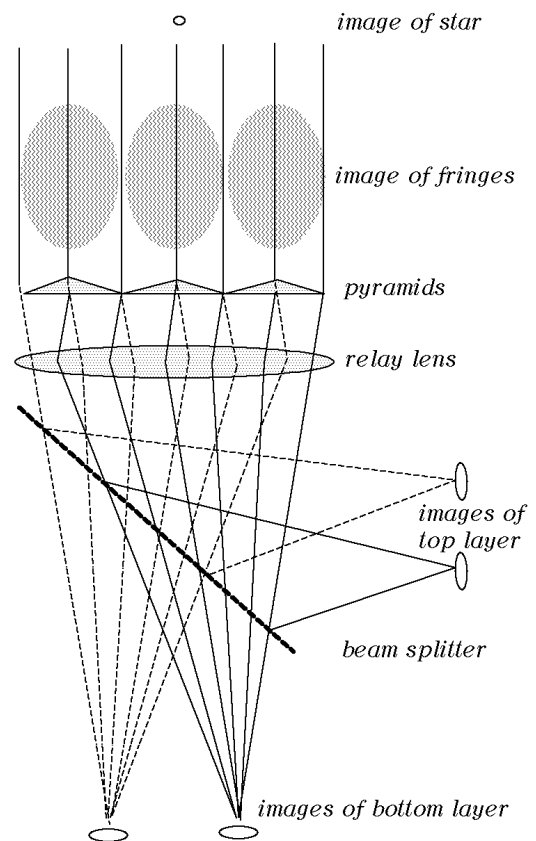


Figure 9. The pyramids break the fringes into four beams (two shown). Two detectors measure each four beams (wave front gradients) when conjugate to the atmospheric layers.

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