

Coulomb blockade- A probe for Non-Abelian Statistics

Roni Ilan

Weizmann Institute of Science

Collaborators:

Ady Stern, Weizmann

Eytan Grosfeld, UIUC

Kareljan Schoutens, Amsterdam

Goals

Suggest a way to detect **non-Abelian quasi-particles**
in quantum Hall systems

In particular: validate the **Read Rezayi construction**
resulting in non-Abelian statistics

What is NA statistics?

Multiply degenerate ground state

Quasi-particle exchange = unitary transformation

$$\Psi_i(R_1, R_2, \dots, R_n) \rightarrow U_{ij} \Psi_j(R_1, R_2, \dots, R_n)$$

Order of exchanges matters

$$U^{2,3}U^{1,2} \neq U^{1,2}U^{2,3}$$

Matrices generally do not commute!

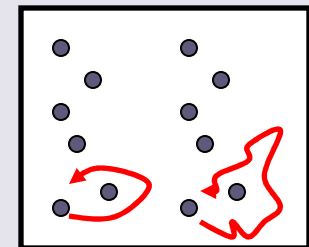
Why detecting it?

- Exotic statistics is still a prediction:

proving the existence non-Abelian statistics would be exciting!

- Switching between states depends on topology of the braid:

potential for a very robust qubit!

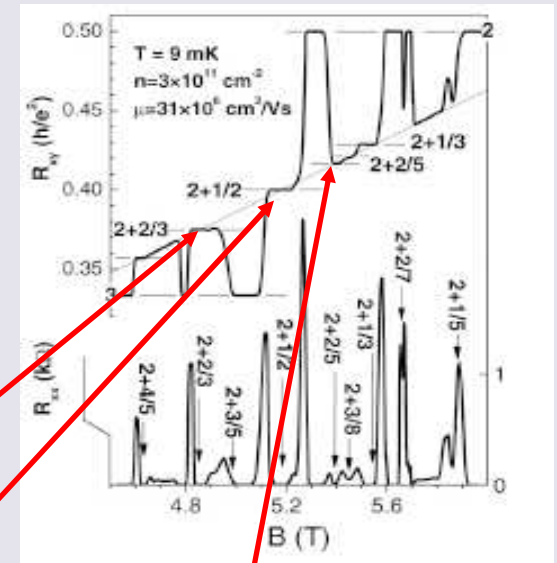


Non-Abelian QH states

$$\nu = 2 + \frac{k}{k+2} \quad \text{Read-Rezayi States (spin polarized)}$$

N. Read and E. Rezayi, PRB (1999).

Theoretically constructed states :
 condensate of clusters of k electrons
 “vortices” with non-Abelian statistics



$k=4$

$k=2$

$k=3$

Some filling fractions match plateaus
 seen in experiment

J. S. Xia et. al. Phys. Rev. Lett. 93, 176809 (2004)

Main Results

Coulomb blockade shows signatures of **clustering**

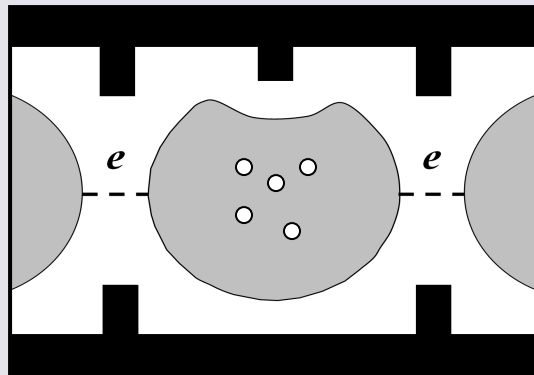
Coulomb blockade can show signatures
of **non-Abelian quasiholes**

Bulk-edge coupling as irreversible relaxation
may eliminate signatures of bulk quasi-holes

Quantum Hall puddles

Finite system – # of electrons quantized to an integer

Transport of charge in resonances of electron tunneling



Finite current when

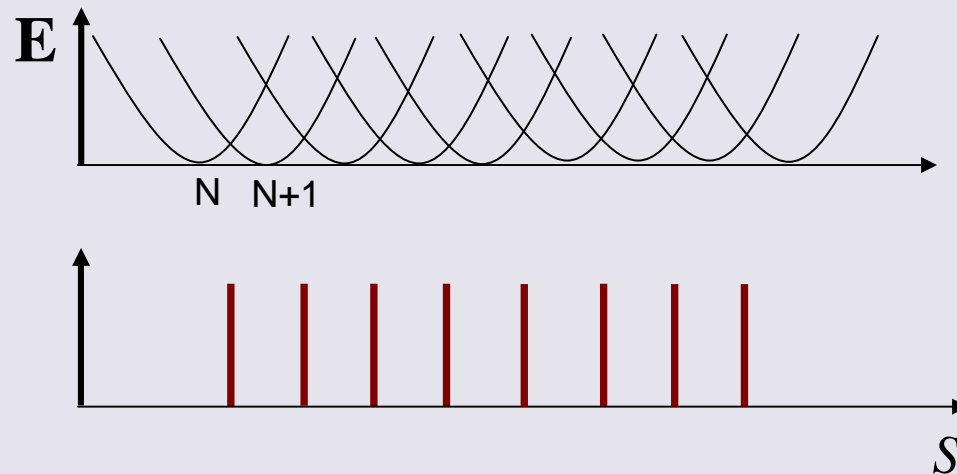
$$E(N_e, n_{qh}, S) = E(N_e + 1, n_{qh}, S)$$

For non-Abelian states the level spacing depends on the number of quasi-holes

Blockade for Laughlin states

Edge theory = Chiral boson

$$E_\varphi = \frac{m}{4\pi} \int dx \left(\partial_x \varphi - 2\pi\nu \frac{B(S - S_0)}{L\phi_0} \right)^2; \quad \partial_x \varphi \propto N/L$$



Equal area separation between consecutive peaks $\Delta S = e/n_0$

Blockade for non-Abelian states

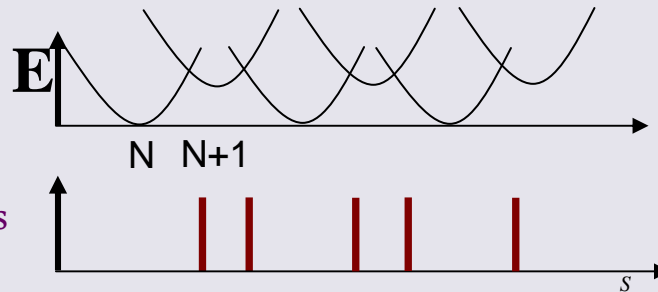
Edge theory = Chiral boson + Z_k parafermions

$$E(N, n_{qh}, S) = E_\varphi(N, S) + E_n(N, n_{qh})$$

$$E_\varphi(N, S) = \frac{\pi v_c}{vL} \left(N - v \frac{B\Delta S}{\phi_0} \right)^2 \quad E_n(N, n_{qh}) = \frac{2\pi v_n}{L} h(N, n_{qh})$$

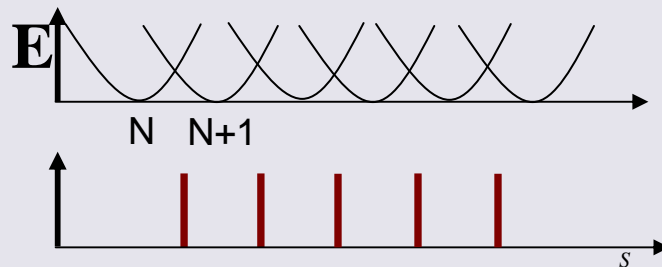
n_{qh} even
 $\psi|0\rangle$

Periodic Boundary Conditions



n_{qh} odd
 $\psi|\sigma\rangle$

Anti - Periodic BC



Majorana fermion

$k=2$

$$\sigma \times \sigma = 1 + \psi$$

$$\psi \times \sigma = \sigma$$

$$\psi \times \psi = 1$$

$$h_\sigma = \frac{1}{16}$$

$$h_\psi = \frac{1}{2}$$

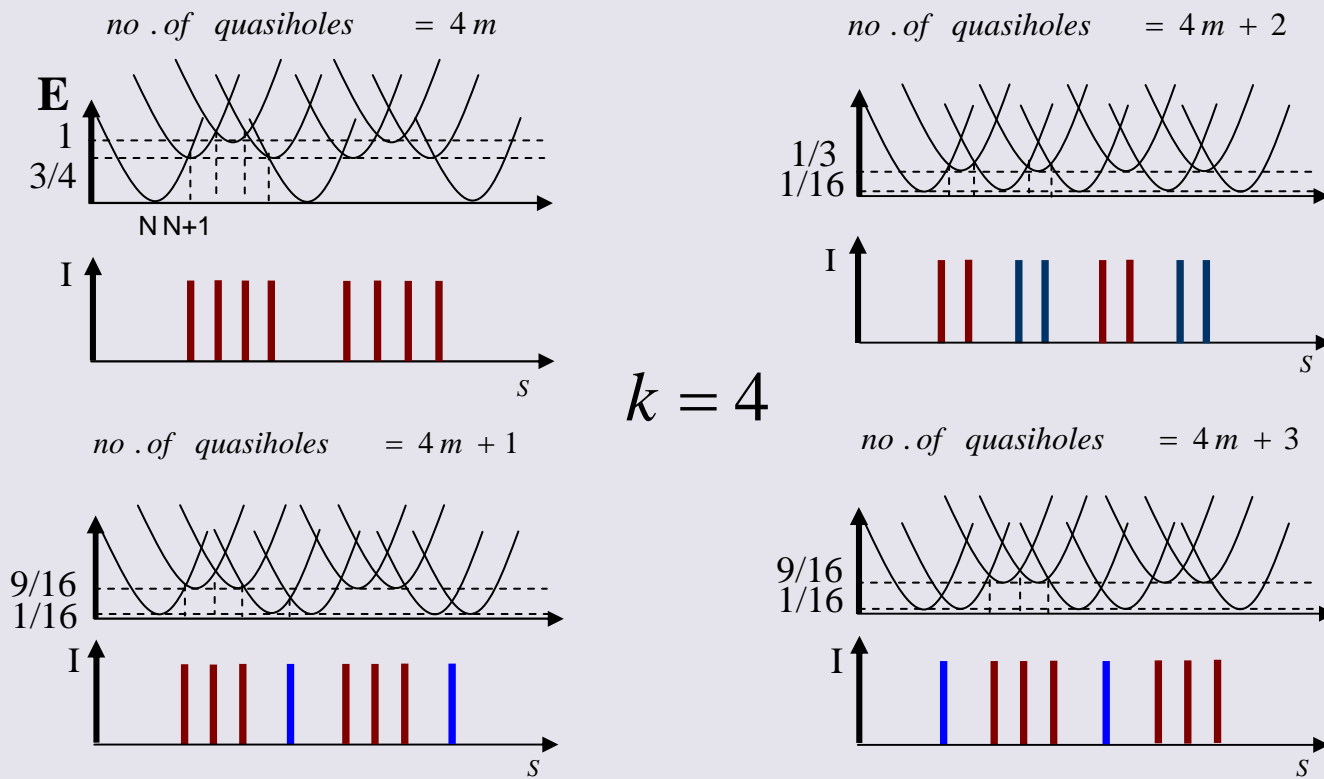
Stern and Halperin, PRL (2006)

Moore and Read, NPB (1991)

K > 2

Majorana fermion replaced by a parafermion

$$\psi^k \sim 1 \quad E_n(N, n_{qh}) = \frac{2\pi v_n}{L} h(\psi^N \sigma_q)$$



Bulk-edge relaxation

Tunneling neutral particles between bulk and edge
changes parafermion edge modes

Irreversible relaxation mechanism

For RR - eliminates dependence on # of quasi-holes

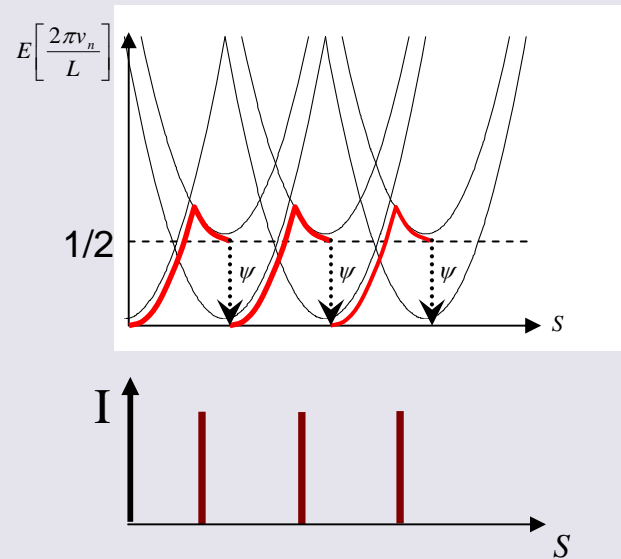
Odd k : $\frac{k-1}{2}$ Peaks, $\frac{k+1}{2}$ Peaks

Even k : $\frac{k}{2}$

Relaxation for $k=2$

Edge state is emptied after a tunneling resonance

$$|0\rangle \xrightarrow{\text{tunneling}} \psi|0\rangle \xrightarrow{\text{decay}} |0\rangle \xrightarrow{\text{tunneling}} \psi|0\rangle$$



$k=2$: no bunching at all

Time scale of experiment $<$ Time scale for relaxation

Results

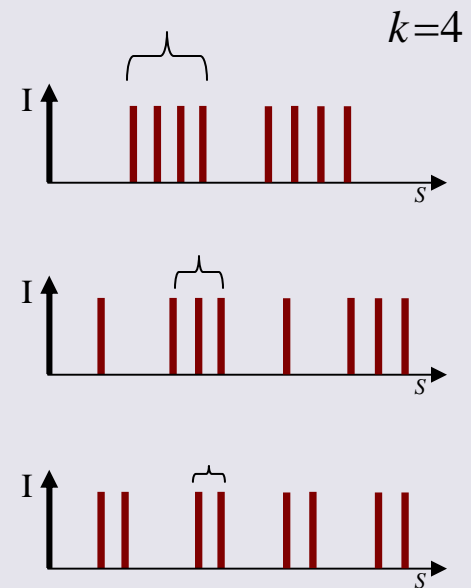
RR states:

Peaks show a bunching pattern

Pattern varies with # of bulk quasi-holes

Pattern has period k – indication for clustering

Bulk-edge coupling eliminates dependence on # of quasi-holes



References: PRL 100 (2008), PRB 79 (2009)