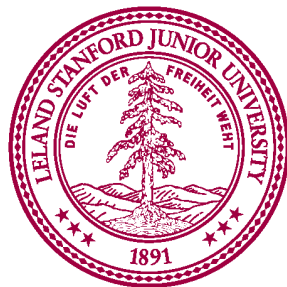


# Enhanced Pairing in the Plaquette-Hubbard Model

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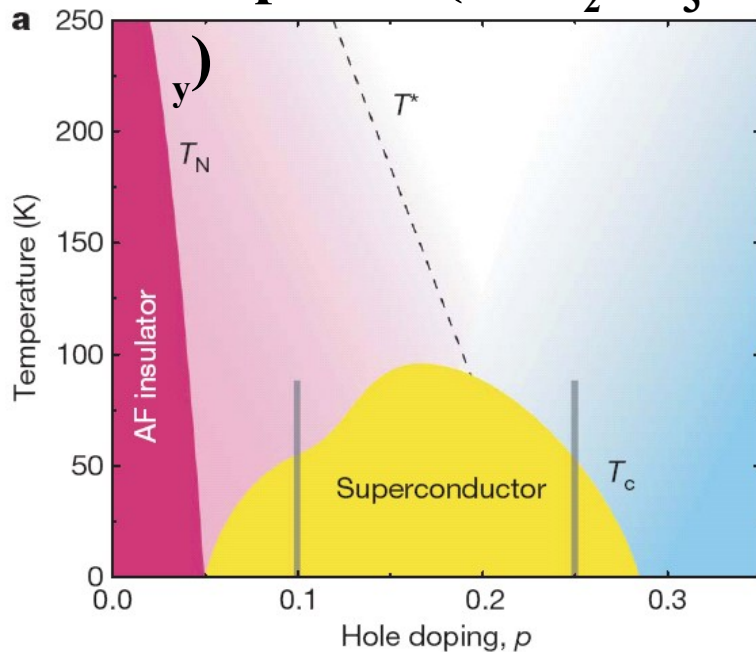
UCIrvine  
University of California, Irvine



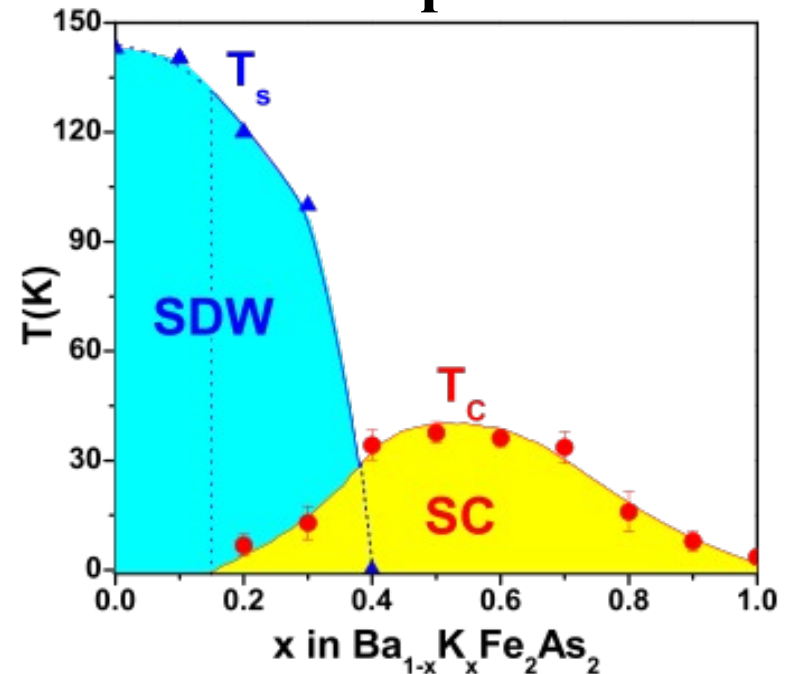
# Background: Pairing from repulsive interactions!

- **Large  $U/W$**  - little room to retardation
- Proximity to **Mott insulator/magnetically ordered states**
- **"Unconventional" order parameter and unusual (non-Fermi liquid) normal state**

Cuprates ( $\text{YBa}_2\text{Cu}_3\text{O}_{7-y}$ )

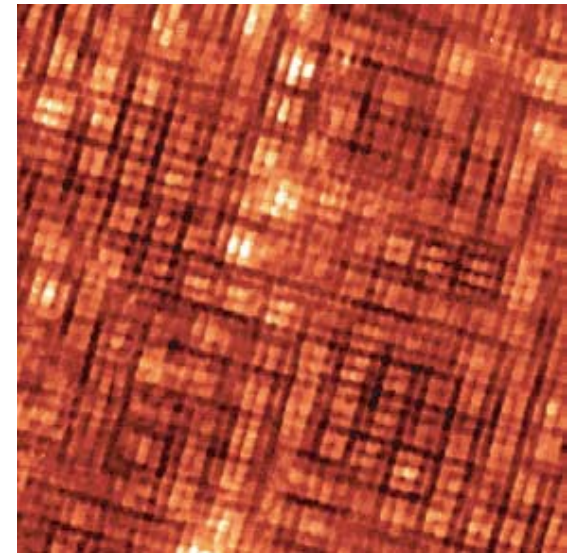
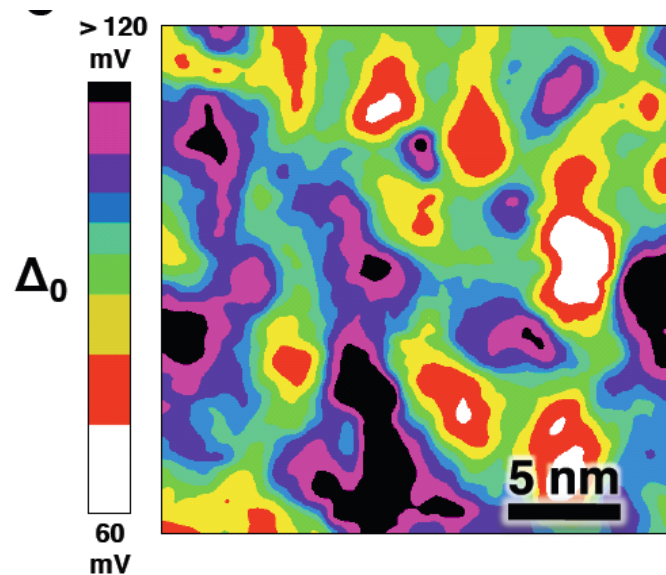


Iron pnictides



# Inhomogeneity in HTS

- **Inhomogeneities** (either extrinsic or intrinsic) are everywhere!
- $T_c$  is surprisingly robust - Short coherence length
- STM experiments:



A. Pushap et al, arXiv (2009)

T. Hanaguri, Nature (2004)

# The problem: Recipe for a high $T_c$ superconductor?

Parts of the answer are known:

- on-site  $U$  can be good, further-range Coulomb bad
- Optimal  $T_c$  - intermediate  $W/U$

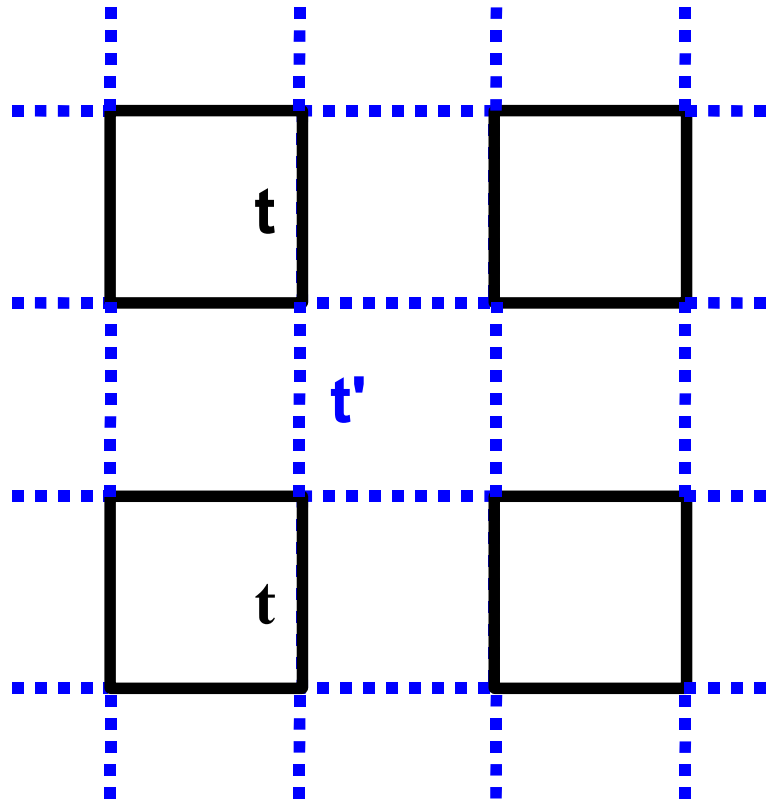
**Inhomogeneity - good/bad?**

Hubbard Hamiltonian

$$H = - \sum_{\langle i,j \rangle, \sigma} t_{ij} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{H.c.}) + \sum_i U_i n_{i,\uparrow} n_{i,\downarrow}$$

**"Rules of the game:"** Vary  $t_{ij}$ ,  $U$ , keeping  $\max\{t_{ij}\} \leq t$ .

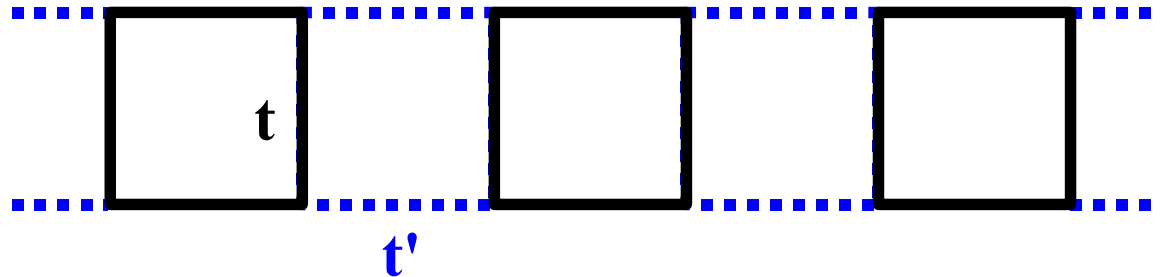
# The Plaquette Hubbard model



- **The single Hubbard plaquette:** d wave-Mott g.s., pair binding for  $U < 4.6t$

Altman, Auerbach (2002), Tsai, Kivelson (2006), Yao et al. (2008),  
Rey et al. (2008), Baruch, Orgad (2009)

# The Plaquette Hubbard ladder



- **2-leg Hubbard ladder**: spin gap, power-law d-SC correlations, amenable to weak coupling RG, numerical methods (e.g. DMRG)
- $T_c = 0!$  (1D)
- Energy scales characterizing pairing:  $\Delta_{s'}$ ,  $\Delta_{pb}$
- Later: the phase stiffness  $\rho_s$

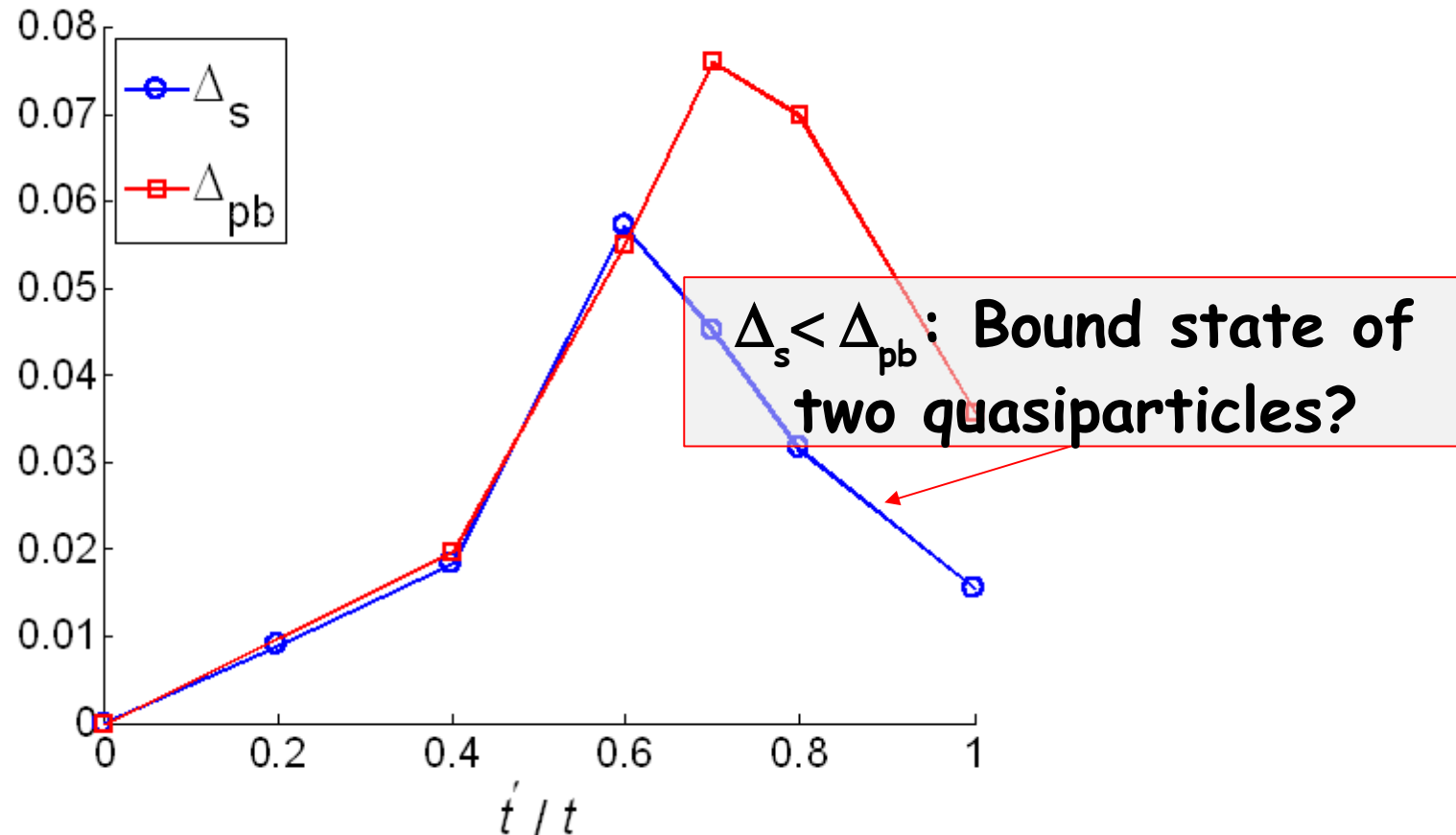
# DMRG results

$$U = 8, n = 0.875$$

$$\Delta_s = (E_{S=1} - E_{S=0}) / 2 \quad \Delta_{pb} = (2E_{N+1} - E_N - E_{N+2}) / 2$$

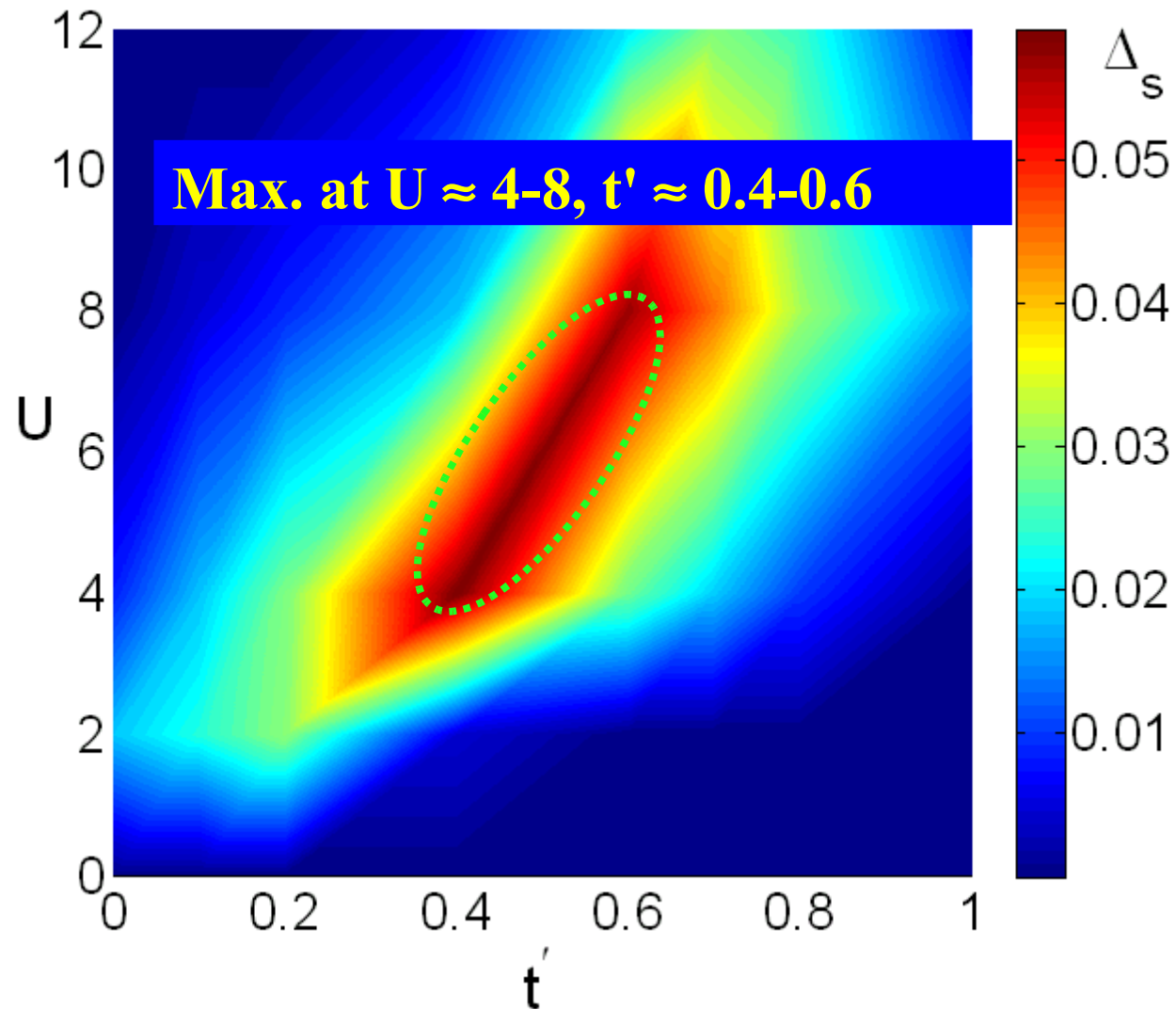
Extrapolated to the  $L \rightarrow \infty$  limit (sizes up to  $64 \times 2$ )

All  
energies in  
units of  $t$



# Results

$\Delta_s$  for  $n = 0.875$  vs.  $t'$ ,  $U$



Maximum pairing for  $t' < 1$  !

What about phase ordering?

# Phase ordering scale

- A rough estimate of the “phase ordering scale”:

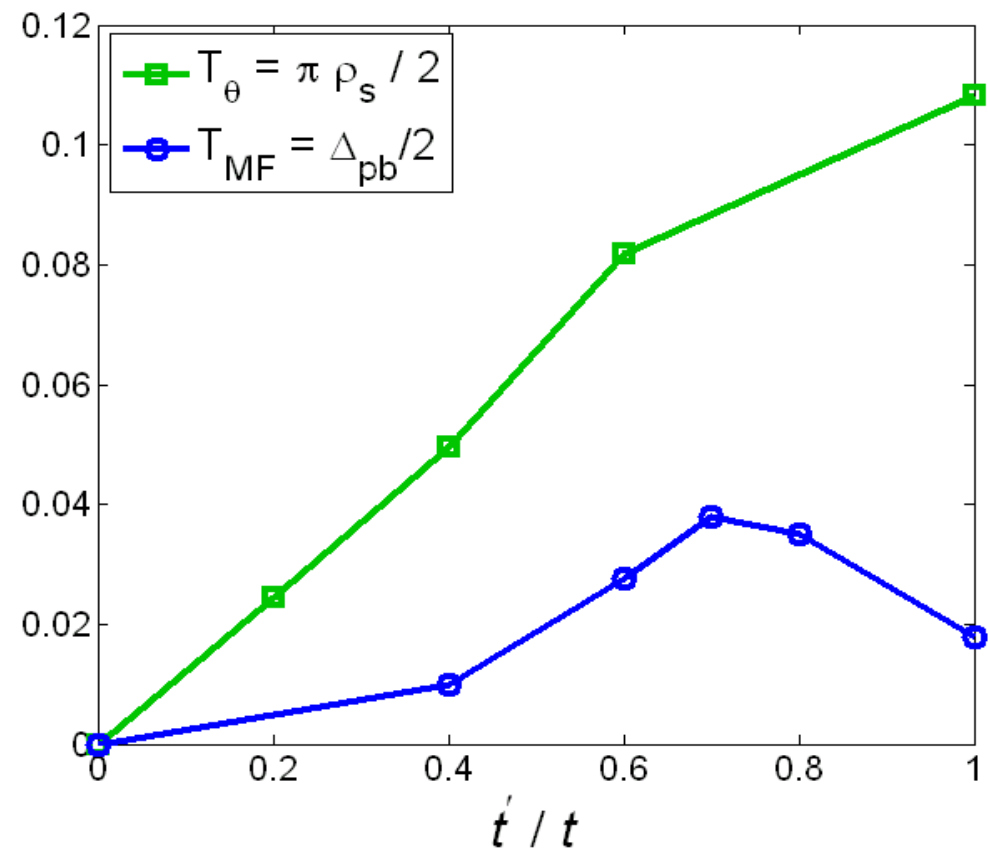
$$T_{\theta} = \pi \rho_s / 2$$

Where  $\rho_s = \partial^2 E / \partial \phi^2 (L/2a)$  (a - lattice constant)

- Compare to the “pairing scale”:

$$T_{MF} = \Delta_{pb} / 2$$

$$T_{\theta} > T_{MF} !$$



# Conclusions

- Pairing in the “**Plaquette Hubbard ladder**”:
  - Optimized pairing for  $t'/t \approx 0.4-0.6$ ,  $U \approx 4-8$
  - **Large phase stiffness**
- What causes the pairing enhancement?
- Other “optimal structures”?